Erratum for "Slope Filtrations Revisited"

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Abstract. Some corrections are given for the manuscript "Slope Filtrations Revisited", Documenta Math., Vol. 10 (2005), 447-525

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Laurent Berger has pointed out that the construction of Teichmüller presentations in [3, Definition 2.5.1] is not valid: it fails to properly account for the nonlinearity of the Teichmüller map. This would appear to invalidate those results of [3] depending on the use of Teichmüller presentations, or on plus-minus-zero presentations. Fortunately, these can be corrected by adapting the technique of strong semiunit decompositions from [2], as follows.

Retain notation as in [3, § 2.5]. A *strong semiunit presentation* of $x \in \Gamma_I$ is a convergent sum $x = \sum_{i \in \mathbb{Z}} u_i \pi^i$ in which:

(a) each nonzero u_i belongs to Γ and satisfies $v_n(u_i) = v_0(u_i)$ for all $n \geq 0$;

(b) if $i > j$ and u_i, u_j are both nonzero, then $v_0(u_i) < v_0(u_j)$.

Such a presentation always exists by the same proof as in [2, Proposition 3.14], but there is no uniqueness property. Nonetheless, in each of [3, Proposition 3.3.7(c), Proposition 4.2.2, Lemma 4.3.2], one may safely replace all references to Teichmüller presentations (including implicit references via plus-minus-zero presentations) with strong semiunit presentations. (One should also disregard the parenthetical remark about canonicality in the proof of [3, Proposition 4.2.2].)

This substitution does not suffice for the proof of surjectivity in [3, Lemma 4.3.1], which uses the uniqueness property of Teichmüller presentations. This is harmless for the rest of the paper, because this lemma is used nowhere. For completeness, we point out that the lemma is an immediate consequence of a result of Fourquaux [1, Corollaire 3.9.19] (applied with $a = 1$).

Ruochuan Liu points out that the proof of [3, Lemma 2.9.1] is incomplete: it is only valid in case f has no slopes in $[s', s)$, as otherwise we cannot choose the unit u in the first sentence of the proof. To complete the proof in general, first note that the existence of g satisfying (a) and (b) follows from [3, Lemma 2.6.7]. To prove (c), choose s'' with $s' < s'' < s$ such that f has no slopes in $[s'', s)$. By the proof of [3, Lemma 2.9.1] as written, f is divisible by g in $\Gamma_{[s'',r]}$. However, since g has no slopes less than s, g is a unit in $\Gamma_{[s',s'']}$, so f is also divisible by g in that ring. Since the intersection $\Gamma_{[s',s'']} \cap \Gamma_{[s'',r]}$ inside $\Gamma_{[s'',s'']}$ is equal to $\Gamma_{[s',r]}$ by [3, Corollary 2.5.7], f is divisible by g in $\Gamma_{[s',r]}$ as desired.
Liu also notes a gap in the proof of [3, Lemma 2.9.3]: it is necessary to ensure that $x_{i+1} \in \Gamma_r[\pi^{-1}]$. To fix this, we must replace $g_{i+1} - x_i$ wherever it appears by some $y_i \in \Gamma_r$ such that $g_{i+1} - x_i - y_i$ is divisible by h_{i+1} in Γ_{i+1}; this can be carried out by an argument similar to [3, Lemma 2.9.2].
We also take this opportunity to point out two errata to [2]. First (as noted by Kevin Buzzard), in the introduction (p. 95), it is incorrectly asserted that "Γ_{con} consists of series which take integral values on some open annulus with outer radius 1." In fact, an element of Γ_{con} acquires this property only after multiplication by a large power of u (and conversely). Second, in [2, Lemma 2.3], R should be taken to be a Bézout domain, not merely a Bézout ring.

References

[1] L. Fourquaux, Logarithme de Perrin-Riou pour des extensions associées à un groupe de Lubin-Tate, PhD thesis, Université de Paris 6, 2005; `http://tel.archives-ouvertes.fr/tel-00011919`.

[2] K.S. Kedlaya, A p-adic local monodromy theorem, *Annals of Math.* 160 (2004), 93–184.

[3] K.S. Kedlaya, Slope filtrations revisited, *Doc. Math.* 10 (2005), 447–525.

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