Erratum for "Tropical Convexity"

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Abstract. Theorem 29 and Corollary 30 of [1] are incorrect. This only concerns the application of tropical convexity to phylogenetic trees. None of the results on tropical convexity itself is affected.

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Theorem 29 and Corollary 30 of [1] are not correct. If D is a symmetric matrix which represents a finite metric, then the tropical polytope P_D always contains Isbell's injective hull of the metric, but in general these two polyhedral spaces are not equal. The flaw lies in the statement (made in the proof of Theorem 29) that for any vertex (y, z) of P_{-D}, the vector y is a column of $-D$. The tropical polytope P_{-D} can have vertices for which this is not the case, corresponding to vertices in the tropical convex hull which are not in the generating set. The injective hull of D is the intersection of P_{-D} with the linear space $\{y = z\}$. If the metric D is a tree metric, then the tropical polytope P_D is one-dimensional and is indeed equal to the given tree, so Theorem 28 is correct as stated.

For instance, for a generic metric D on four points, the tropical tetrahedron P_D given by the tropical convex hull of the negated columns of the matrix is three-dimensional, while the injective hull is a two-dimensional complex consisting of four edges emanating from a quadrangle [2, Figure A3]. Even in this case, there does not seem be a straightforward relationship between the combinatorial structure of the tropical tetrahedron P_D and that of the injective hull of D.

While the connection between metrics and regular subdivisions of products of simplices via tropical convex hulls is invalid, metrics are intimately related to subdivisions of other polytopes, namely hypersimplices, as shown in [3].

References

[1] M. Develin and B. Sturmfels: Tropical Convexity, *Documenta Math.* 9 (2004) 1–27.

[2] A. Dress: Trees, Tight Extensions of Metric Spaces, and the Cohomological Dimension of Certain Groups, *Advances in Mathematics* 53 (1984) 321–402.

[3] B. Sturmfels and J. Yu: Classification of Six-Point Metrics, *Electronic Journal of Combinatorics* 11 (2004/05) R44.

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