Asymptotics of Complete Kähler-Einstein Metrics – Negativity of the Holomorphic Sectional Curvature

Georg Schumacher

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Abstract. We consider complete Kähler-Einstein metrics on the complements of smooth divisors in projective manifolds. The estimates proven earlier by the author [5] imply that in directions parallel to the divisor at infinity the metric tensor converges to the Kähler-Einstein metric on the divisor. Here we show that the holomorphic sectional curvature is bounded from above by a negative constant near infinity.

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1. Introduction and Statement of the Result

Complete Kähler-Einstein metrics ds^2_X of constant Ricci curvature on quasi-projective varieties are of interest in various geometric situations. The existence of a unique complete Kähler-Einstein metric ds^2_X of constant Ricci curvature -1 on a manifold X of the form $\overline{X}\backslash C$, where $\overline{X}$ is a compact complex manifold and C a smooth divisor is guaranteed under the condition

$$K_{\overline{X}} + C > 0 \qquad (N)$$

(cf. [2, 3, 7]).

In [5], asymptotic properties of ds^2_X (with Kähler form ω_X) were investigated. In the special asymptotic situation, it is possible to prove estimates for the curvature tensor based on constant negative Ricci curvature. We obtain the following theorem.

Theorem 1. *Let $\overline{X}$ be a compact complex surface, $C \subset \overline{X}$ a smooth divisor satisfying (N). Then the holomorphic sectional curvature of the complete Kähler-Einstein metric on $X = \overline{X} \backslash C$ is bounded from above by a negative constant near the compactifying divisor. The sectional and holomorphic bisectional curvatures are bounded on X.*

For $X = \mathbb{P}_2$, the assumption of the theorem is satisfied, if the degree of the curve C is at least four. So the statement of the above theorem appears to be related to the Kobayashi conjecture about the algebraic degeneracy of entire holomorphic curves and the hyperbolicity of complements of plane curves.

2. Properties of the Complete Kähler-Einstein Metric

The above condition (N) also implies the existence of a Kähler-Einstein metric ω_C on C. Using a canonical section σ of $[C]$ as a local coordinate function on $\overline{X}$ one can restrict the complete Kähler-Einstein metric ω_X to the locally defined sets $C_{\sigma_0} = \{\sigma = \sigma_0\}$. The notion of locally uniform convergence of $\omega_X | C_{\sigma_0}$, where $\sigma_0 \to 0$, makes sense.

Theorem 2 ([5]). *The Kähler-Einstein metric ω_X converges to the Kähler-Einstein metric ω_C, when restricted to directions parallel to C.*

The result is a precise analytic version of the adjunction formula: Let h be an hermitian metric on $[C]$ and Ω^∞ a volume form of class C^∞ on $\overline{X}$, such that the restriction of Ω^∞ / h to C is the Kähler-Einstein volume form on C.
In terms of the Hölder spaces $C^{k,\lambda}(X)$ and $C^{k,\lambda}(W)$ for open subsets $W \subset X$ depending on quasi-coordinates used in [1] by Cheng and Yau (cf. [2, 3]), the above statement follows from the estimate below [5]: *There exists a number $0 < \alpha \leq 1$ such that for all $k \in \mathbb{N}$ and all $0 < \lambda < 1$ the volume form of the complete Kähler-Einstein metric is of the form*

$$\frac{2\Omega^\infty}{\|\sigma\|^2 \log^2(1/\|\sigma\|^2)} \left(1 + \frac{\nu}{\log^\alpha(1/\|\sigma\|^2)} \right) \text{ with } \nu \in C^{k,\lambda}(W) \tag{1}$$

For any given $\nu \in C^{k,\lambda}(W)$ according to [5, 2] there exists some $\mu \in C^{k-1,\lambda}(W)$ such that

$$\frac{\partial \nu}{\partial \sigma} = \frac{\mu}{\sigma \log(1/|\sigma|^2)}.$$

From now on we assume that $\dim_{\mathbb{C}}(X) = 2$.

Let (σ, w) be local coordinates near $[C]$, and $g_{\sigma\overline{\sigma}}, g_{\sigma\overline{w}}$ etc. the coefficients of the metric tensor of the Kähler-Einstein metric ω_X.

$$g_{\sigma\overline{\sigma}} = \frac{2}{|\sigma|^2 \log^2(1/|\sigma|^2)} \left(1 + \frac{g^0_{\sigma\overline{\sigma}}}{\log^\alpha(1/|\sigma|^2)}\right) \tag{2}$$

$$g_{\sigma\overline{w}} = \frac{g^0_{\sigma\overline{w}}}{\sigma \log^{1+\alpha}(1/|\sigma|^2)} \tag{3}$$

$$g_{w\overline{\sigma}} = \frac{g^0_{w\overline{\sigma}}}{\overline{\sigma} \log^{1+\alpha}(1/|\sigma|^2)} \tag{4}$$

$$g_{w\overline{w}} = g^\infty_{w\overline{w}} \left(1 + \frac{g^0_{w\overline{w}}}{\log^\alpha(1/|\sigma|^2)}\right) \tag{5}$$

where $g^0_{\sigma\overline{\sigma}}$, $g^0_{\sigma\overline{w}}$, $g^0_{w\overline{\sigma}}$, $g^0_{w\overline{w}}$ are in $C^{k,\lambda}(W)$, whereas $g^\infty_{w\overline{w}}$ is of class C^∞, and $\omega_C = \sqrt{-1}(g^\infty_{w\overline{w}}|C)dw \wedge \overline{dw}$ has constant curvature -1. We observe that in the determinant of the components of the metric tensor the diagonal terms dominate the rest. Moreover:

PROPOSITION 1.

$$g^{\overline{\sigma}\sigma} \sim |\sigma|^2 \log^2(1/|\sigma|^2) \tag{6}$$

$$g^{\overline{\sigma}w}, g^{\overline{w}\sigma} = O\left(|\sigma| \log^{1-\alpha}(1/|\sigma|^2)\right) \tag{7}$$

$$g^{\overline{w}w} \sim 1 \tag{8}$$

where $p \sim q$ denotes the existence of some $C > 1$ such that $(1/C) \cdot p \leq q \leq C \cdot p$.

3. ASYMPTOTICS OF THE CURVATURE TENSOR

The order of the arguments is critical. We begin with the off-diagonal terms of the curvature tensor, which require special attention. In the sequel we need the volume form Ω_X in our local coordinates (σ, w), and we set $D := g_{\sigma\overline{\sigma}} \cdot g_{w\overline{w}} - g_{\sigma\overline{w}} \cdot g_{w\overline{\sigma}}$, i.e.

(9)

$$D = \frac{2g^\infty_{w\overline{w}}}{|\sigma|^2 \log^2(1/|\sigma|^2)} \left(1 + \frac{g^0_{\sigma\overline{\sigma}} + g^0_{w\overline{w}}}{\log^\alpha(1/|\sigma|^2)} + \frac{g_{\sigma\overline{\sigma}} \cdot g_{w\overline{w}} - (g^0_{\sigma\overline{w}} \cdot g^0_{w\overline{\sigma}}/2g^\infty_{w\overline{w}})}{\log^{2\alpha}(1/|\sigma|^2)}\right).$$

We estimate

$$R_{\sigma\overline{w}\sigma\overline{w}} = O\left(1/\log^{2+\alpha}(1/|\sigma|^2)\right) \tag{10}$$

Proof of (10). We compute $-D \cdot R_{\sigma\overline{w}\sigma\overline{w}} = D \cdot \frac{\partial^2 g_{\sigma\overline{w}}}{\partial\sigma\partial\overline{w}} - \frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma} g_{w\overline{w}} \frac{\partial g_{\sigma\overline{w}}}{\partial\overline{w}} + \frac{\partial g_{\sigma\overline{w}}}{\partial\sigma} g_{w\overline{\sigma}} \frac{\partial g_{\sigma\overline{w}}}{\partial\overline{w}} + \frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma} g_{\sigma\overline{w}} \frac{\partial g_{w\overline{w}}}{\partial\overline{w}} - \frac{\partial g_{\sigma\overline{w}}}{\partial\sigma} g_{\sigma\overline{\sigma}} \frac{\partial g_{w\overline{w}}}{\partial\overline{w}}$. We gather the first three terms: $D \cdot \frac{\partial^2 g_{\sigma\overline{w}}}{\partial\sigma\partial\overline{w}} = \frac{-2g^\infty_{w\overline{w}} \cdot \frac{\partial g^0_{\sigma\overline{w}}}{\partial\overline{w}}}{|\sigma|^2\sigma^2 \log^{3+\alpha}(1/|\sigma|^2)} \cdot \left(1 + \frac{g^0_{\sigma\overline{\sigma}} + g^0_{w\overline{w}}}{\log^\alpha(1/|\sigma|^2)} + \frac{g_{\sigma\overline{\sigma}} \cdot g_{w\overline{w}} - (g^0_{\sigma\overline{w}} \cdot g^0_{w\overline{\sigma}}/2g^\infty_{w\overline{w}})}{\log^{2\alpha}(1/|\sigma|^2)}\right) \cdot \left(1 + O\left(1/\log^1(1/|\sigma|^2)\right)\right)$. Next $-\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma} g_{w\overline{w}} \frac{\partial g_{\sigma\overline{w}}}{\partial\overline{w}} = \frac{-2g^\infty_{w\overline{w}} \cdot \frac{\partial g^0_{\sigma\overline{w}}}{\partial\overline{w}}}{|\sigma|^2\sigma^2 \log^{3+\alpha}(1/|\sigma|^2)} \cdot$

$\left(1+\frac{g^0_{\sigma\overline{\sigma}}+g^0_{w\overline{w}}}{\log^\alpha(1/|\sigma|^2)}+\frac{g^0_{\sigma\overline{\sigma}}\cdot g^0_{w\overline{w}}}{\log^{2\alpha}(1/|\sigma|^2)}\right)\cdot\left(1+O\left(1/\log^1(1/|\sigma|^2)\right)\right)$, and $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}g_{w\overline{\sigma}}\frac{\partial g_{\sigma\overline{w}}}{\partial w}=$ $\frac{-g_{\sigma\overline{\sigma}}\cdot g_{w\overline{w}}\cdot\frac{\partial g^0_{\sigma\overline{w}}}{\partial w}}{|\sigma|^2\sigma^2\log^{3+\alpha}(1/|\sigma|^2)}\cdot\left(1+O\left(1/\log^1(1/|\sigma|^2)\right)\right)$.

Hence the sum of the first three terms is of the form $O\left(1/|\sigma|^4\log^{4+3\alpha}(1/|\sigma|^2)\right)$. Concerning the sum of the last two terms $\left(\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}\cdot g_{\sigma\overline{w}}-\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}\cdot g_{\sigma\overline{\sigma}}\right)\cdot\frac{\partial g_{w\overline{w}}}{\partial w}$ we observe that both $\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}\cdot g_{\sigma\overline{w}}$, and $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}\cdot g_{\sigma\overline{\sigma}}$ are of the form

$\frac{-2g^0_{\sigma\overline{w}}}{\sigma^2|\sigma|^2\log^{3+\alpha}(1/|\sigma|^2)}\cdot\left(1+\frac{g^0_{\sigma\overline{\sigma}}}{\log^\alpha(1/|\sigma|^2)}\right)\cdot\left(1+O\left(1/\log^1(1/|\sigma|^2)\right)\right)$.

Hence again the sum is of order $O\left(1/|\sigma|^4\log^{4+3\alpha}(1/|\sigma|^2)\right)$. □

Next, we claim

$$R_{\sigma\overline{\sigma}w\overline{w}}=O\left(1/\log^{2+\alpha}(1/|\sigma|^2)\right) \tag{11}$$

Proof. We compute $-D\cdot R_{\sigma\overline{\sigma}w\overline{w}}=D\cdot\frac{\partial^2 g_{\sigma\overline{\sigma}}}{\partial w\partial\overline{w}}-\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}g_{w\overline{w}}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}}+\frac{\partial g_{\sigma\overline{w}}}{\partial w}g_{w\overline{\sigma}}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}}+\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}g_{\sigma\overline{w}}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial w}}-\frac{\partial g_{\sigma\overline{w}}}{\partial w}g_{\sigma\overline{\sigma}}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial w}}$. It follows immediately that all summands are of the class $O\left(1/|\sigma|\log^{4+\alpha}(1/|\sigma|^2)\right)$. □

The remaining estimates are shown in several cycles.

Step 1. *The following estimates hold for the components of the curvature tensor:*

$$R_{\sigma\overline{\sigma}\sigma\overline{\sigma}}=O\left(1/|\sigma|^4\log^2(1/|\sigma|^2)\right) \tag{12}$$

$$R_{\sigma\overline{\sigma}\sigma\overline{w}}=O\left(1/|\sigma|^3\log^2(1/|\sigma|^2)\right) \tag{13}$$

$$R_{\sigma\overline{w}w\overline{w}}=O(1/|\sigma|\log^{1+\alpha}(1/|\sigma|^2)) \tag{14}$$

$$R_{w\overline{w}w\overline{w}}=O(1) \tag{15}$$

Proof. For (12) we estimate $\frac{\partial^2 g_{\sigma\overline{\sigma}}}{\partial\sigma\partial\overline{\sigma}}$, $\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}g^{\overline{\sigma}\sigma}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}}=O\left(1/|\sigma|^4\log^2(1/|\sigma|^2)\right)$, $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}g^{\overline{w}\sigma}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}}$, $\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}g^{\overline{\sigma}w}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial\sigma}}=O\left(1/|\sigma|^4\log^{2+2\alpha}(1/|\sigma|^2)\right)$, $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}g^{\overline{w}w}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial\sigma}}=O\left(1/|\sigma|^4\log^{1+\alpha}(1/|\sigma|^2)\right)$.

We consider (13): $\frac{\partial^2 g_{\sigma\overline{\sigma}}}{\partial\sigma\partial\overline{w}}$, $\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}g^{\overline{\sigma}\sigma}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}}=O(1/|\sigma|^3\log^2(1/|\sigma|^2))$, $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}g^{\overline{w}\sigma}\overline{\frac{\partial g_{\sigma\overline{\sigma}}}{\partial w}}$, $\frac{\partial g_{\sigma\overline{\sigma}}}{\partial\sigma}g^{\overline{\sigma}w}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial w}}$, $\frac{\partial g_{\sigma\overline{w}}}{\partial\sigma}g^{\overline{w}w}\overline{\frac{\partial g_{w\overline{\sigma}}}{\partial w}}=O(1/|\sigma|^3\log^{2+2\alpha}(1/|\sigma|^2))$

In the same way we arrive at the estimates (14), (15). □

Some of these estimates need to be improved in a second step.

Step 2.

$$R_{\sigma\overline{\sigma}\sigma\overline{\sigma}}=O(1/|\sigma|^4\log^{3+\alpha}(1/|\sigma|^2)) \tag{16}$$

$$R_{\sigma\overline{\sigma}\sigma\overline{w}}=O(1/|\sigma|^3\log^{3+\alpha}(1/|\sigma|^2)) \tag{17}$$

Proof. Concerning (16) we consider the equation

$$-g_{\sigma\overline{\sigma}} = R_{\sigma\overline{\sigma}\sigma\overline{\sigma}}g^{\overline{\sigma}\sigma} + R_{\sigma\overline{\sigma}\sigma\overline{w}}g^{\overline{w}\sigma} + R_{\sigma\overline{\sigma}w\overline{\sigma}}g^{\overline{\sigma}w} + R_{\sigma\overline{\sigma}w\overline{w}}g^{\overline{w}w}$$

According to Step 1 and Proposition 1 the term $R_{\sigma\overline{\sigma}\sigma\overline{\sigma}}g^{\overline{\sigma}\sigma}$ can be estimated from above and below by $1/|\sigma|^2$ whereas the remaining terms are at least of the class $O\left(1/|\sigma|^2\log^{1+\alpha}(1/|\sigma|^2)\right)$. This proves (16).
Next

$$-g_{\sigma\overline{w}} = R_{\sigma\overline{w}\sigma\overline{\sigma}}g^{\overline{\sigma}\sigma} + R_{\sigma\overline{w}\sigma\overline{w}}g^{\overline{w}\sigma} + R_{\sigma\overline{w}w\overline{\sigma}}g^{\overline{\sigma}w} + R_{\sigma\overline{w}w\overline{w}}g^{\overline{w}w}$$

is of the class $O\left(1/|\sigma|\log^{1+\alpha}(1/|\sigma|^2)\right)$, and on the right-hand side all terms but the first are a priori at least in $O\left(1/|\sigma|\log^{1+\alpha}(1/|\sigma|^2)\right)$, whereas $R_{\sigma\overline{w}\sigma\overline{\sigma}}g^{\overline{\sigma}\sigma}$ so far is in $O\left(1/|\sigma|\right)$. This shows (17). □
We need to do (16) again.

Step 3.

$$R_{\sigma\overline{\sigma}\sigma\overline{\sigma}} = O\left(1/|\sigma|^4\log^4(1/|\sigma|^2)\right) \tag{18}$$

We consider once again

$$-g^2_{\sigma\overline{\sigma}} = R_{\sigma\overline{\sigma}\sigma\overline{\sigma}}g^{\overline{\sigma}\sigma}g_{\sigma\overline{\sigma}} + R_{\sigma\overline{\sigma}\sigma\overline{w}}g^{\overline{w}\sigma}g_{\sigma\overline{\sigma}} + R_{\sigma\overline{\sigma}w\overline{\sigma}}g^{\overline{\sigma}w}g_{\sigma\overline{\sigma}} + R_{\sigma\overline{\sigma}w\overline{w}}g^{\overline{w}w}g_{\sigma\overline{\sigma}}.$$

The last three terms on the right-hand side are at least in $O\left(1/|\sigma|^4\log^{4+\alpha}(1/|\sigma|^2)\right)$, whereas $g^2_{\sigma\overline{\sigma}}$ is in $O\left(1/|\sigma|^4\log^4(1/|\sigma|^2)\right)$. This shows (18), and moreover $-R_{\sigma\overline{\sigma}\sigma\overline{\sigma}} \sim g^2_{\sigma\overline{\sigma}}$. □

Let us conclude with a refinement of (15)

Step 4.

$$-R_{w\overline{w}w\overline{w}} = g^2_{w\overline{w}}\left(1 + O\left(1/\log^\alpha(1/|\sigma|^2)\right)\right)$$

Proof. We regard $-g_{w\overline{w}} = R_{w\overline{w}w\overline{w}}g^{\overline{w}w} + R_{w\overline{\sigma}w\overline{w}}g^{\overline{\sigma}w} + R_{w\overline{w}\sigma\overline{w}}g^{\overline{w}\sigma} + R_{w\overline{\sigma}\sigma\overline{w}}g^{\overline{\sigma}\sigma}$. The first summand is in $O(1)$, whereas the remaining three terms are at least in $O\left(1/\log^\alpha(1/|\sigma|^2)\right)$. □

We summarize our estimates in the following way.

Proposition 2. *In a neighborhood of the divisor at infinity we have*

$$-R_{\sigma\overline{\sigma}\sigma\overline{\sigma}} = g^2_{\sigma\overline{\sigma}}(1 + O\left(1/\log^\alpha(1/|\sigma|^2)\right)) \tag{19}$$

$$-R_{w\overline{w}w\overline{w}} = g^2_{w\overline{w}}(1 + O\left(1/\log^\alpha(1/|\sigma|^2)\right)) \tag{20}$$

$$R_{\sigma\overline{\sigma}\sigma\overline{w}} = O\left(1/|\sigma|^3\log^{3+\alpha}(1/|\sigma|^2)\right) \tag{21}$$

$$R_{\sigma\overline{\sigma}w\overline{w}} = O\left(1/|\sigma|^2\log^{2+\alpha}(1/|\sigma|^2)\right) \tag{22}$$

$$R_{\sigma\overline{w}\sigma\overline{w}} = O\left(1|\sigma|^2\log^{2+\alpha}(1/|\sigma|^2)\right) \tag{23}$$

$$R_{\sigma\overline{w}w\overline{w}} = O\left(1/|\sigma|\log^{1+\alpha}(1/|\sigma|^2)\right) \tag{24}$$

Proof of Theorem 1. The above Proposition 2 implies that the curvature tensor is dominated by the diagonal terms (19) and (20). We determine a domain of negative holomorphic sectional curvature. Let

$$t = a \cdot |\sigma| \log(1/|\sigma|^2) \frac{\partial}{\partial \sigma} + b \cdot \frac{\partial}{\partial w}$$

with $a, b \in \mathbb{C}$. Then

$$\begin{aligned} \|t\|^2 &= g_{\sigma\overline{\sigma}} |a|^2 \log^2(1/|\sigma|^2) + (g_{\sigma\overline{w}} a\overline{b} + g_{w\overline{\sigma}} b\overline{a})|\sigma| \log(1/|\sigma|^2) + g_{w\overline{w}}|b|^2 \\ &= 2|a|^2 + g_{w\overline{w}}|b|^2 + (|a|^2 + |b|^2) \cdot O\left(1/\log^\alpha(1/|\sigma|^2)\right). \end{aligned}$$

According to Proposition 2 we have

$$-R(t, \overline{t}, t, \overline{t}) = 4|a|^4 + |b|^4 + (|a|^2 + |b|^2)^2 \cdot O\left(1/\log^\alpha(1/|\sigma|^2)\right)$$

Now we pick a small upper bound for $\|\sigma\|^2$ which yields a negative upper bound for the holomorphic sectional curvature. The sectional and holomorphic bisectional curvatures are dealt with in a similar way. □

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Georg Schumacher
Fachbereich Mathematik
Philipps-Universität
D-35032 Marburg
Germany
schumac@mathematik.uni-marburg.de