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*Existence of nonnegative periodic solutions in neutral integro-differential equations with functional delay*

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**Abstract:** The fixed point theorem of Krasnoselskii and the concept of large contractions are employed to show the existence of a periodic solution of a nonlinear integro-differential equation with variable delay

$$x'(t) = - \int_{t-\tau(t)}^t a(t,s)g(x(s)) ds + \frac{d}{dt}Q(t, x(t-\tau(t))) + G(t, x(t), x(t-\tau(t))).$$

We transform this equation and then invert it to obtain a sum of two mappings one of which is completely continuous and the other is a large contraction. We choose suitable conditions for  $\tau$ ,  $g$ ,  $a$ ,  $Q$  and  $G$  to show that this sum of mappings fits into the framework of a modification of Krasnoselskii's theorem so that existence of nonnegative T-periodic solutions is concluded.

**Keywords:** Krasnoselskii's fixed points; periodic solution; large contraction

**AMS Subject Classification:** 34K20, 34K30, 34K40

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