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A remark on the approximation theorems of Whitney and Carleman-Scheinberg

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Abstract: We show that a C^k-smooth mapping on an open subset of $\mathbb{R}^n$, $k \in \mathbb{N} \cup \{0, \infty\}$, can be approximated in a fine topology and together with its derivatives by a restriction of a holomorphic mapping with explicitly described domain. As a corollary we obtain a generalisation of the Carleman-Scheinberg theorem on approximation by entire functions.

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