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On the metric reflection of a pseudometric space in ZF

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Abstract: We show: (i) The countable axiom of choice **CAC** is equivalent to each one of the statements: (a) a pseudometric space is sequentially compact iff its metric reflection is sequentially compact, (b) a pseudometric space is complete iff its metric reflection is complete. (ii) The countable multiple choice axiom **CMC** is equivalent to the statement: (a) a pseudometric space is Weierstrass-compact iff its metric reflection is Weierstrass-compact. (iii) The axiom of choice **AC** is equivalent to each one of the statements: (a) a pseudometric space is Alexandroff-Urysohn compact iff its metric reflection is Alexandroff-Urysohn compact, (b) a pseudometric space **X** is Alexandroff-Urysohn compact iff its metric reflection is ultrafilter compact. (iv) We show that the statement "The preimage of an ultrafilter extends to an ultrafilter" is not a theorem of **ZFA**.

Keywords: weak axioms of choice; pseudometric spaces; metric reflections; complete metric and pseudometric spaces; limit point compact; Alexandroff-Urysohn compact; ultrafilter compact; sequentially compact
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