

Jimmie Lawson

Clifford algebras, Möbius transformations, Vahlen matrices, and B-loops

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Abstract: In this paper we show that well-known relationships connecting the Clifford algebra on negative euclidean space, Vahlen matrices, and Möbius transformations extend to connections with the Möbius loop or gyrogroup on the open unit ball B in n -dimensional euclidean space \mathbb{R}^n . One notable achievement is a compact, convenient formula for the Möbius loop operation $a * b = (a + b)(1 - ab)^{-1}$, where the operations on the right are those arising from the Clifford algebra (a formula comparable to $(w + z)(1 + \bar{w}z)^{-1}$ for the Möbius loop multiplication in the unit complex disk).

Keywords: Bruck loop, Clifford algebra, gyrogroup, Möbius transformations, Vahlen matrices, involutive group

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