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On the combinatorics of Kac's asymmetry function

Abstract: We use categories to recast the combinatorial theory of full heaps, which are certain labelled partially ordered sets that we introduced in previous work. This gives rise to a far simpler set of definitions, which we use to outline a combinatorial construction of the so-called loop algebras associated to affine untwisted Kac–Moody algebras. The finite convex subsets of full heaps are equipped with a statistic called parity, and this naturally gives rise to Kac's asymmetry function. The latter is a key ingredient in understanding the (integer) structure constants of simple Lie algebras with respect to certain Chevalley bases, which also arise naturally in the context of heaps.

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