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Overlapping latin subsquares and full products

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Abstract: We derive necessary and sufficient conditions for there to exist a latin square of order n containing two subsquares of order a and b that intersect in a subsquare of order c. We also solve the case of two disjoint subsquares. We use these results to show that: (a) A latin square of order n cannot have more than $\frac{n}{m}\binom{n}{h}/\binom{m}{h}$ subsquares of order m, where $h = \lceil (m+1)/2 \rceil$. Indeed, the number of subsquares of order m is bounded by a polynomial of degree at most $\sqrt{2m}+2$ in n. (b) For all $n \geq 5$ there exists a loop of order n in which every element can be obtained as a product of all n elements in some order and with some bracketing.

Keywords: latin square, latin subsquare, overlapping latin subsquares, full product in loops
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