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Hopf Structure for Poisson Enveloping Algebras

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Abstract. This work is to obtain a natural Hopf structure of the Poisson enveloping algebra $U(A)$ for a Poisson Hopf algebra A.

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Assume throughout the paper that k denotes a field of characteristic zero. Recall that $A = (A, \cdot, \{\cdot, \cdot\})$ is said to be a Poisson algebra if $(A, \cdot)$ is a commutative k-algebra and $(A, \{\cdot, \cdot\})$ is a Lie algebra such that

$$\{ab, c\} = a\{b, c\} + b\{a, c\}$$

for all $a, b, c \in A$. For every Poisson algebra A, there exists a unique Poisson enveloping algebra $U(A)$, which is a (associative) k-algebra, such that a k-vector space M is a Poisson A-module if and only if M is a $U(A)$-module (see [4, **1**, **5** and **6**]). The main purpose of this paper is to see that if A is also a Hopf algebra with Hopf structure compatible with the given Poisson structure (in this case, A is called a Poisson Hopf algebra) then $U(A)$ is a Hopf algebra.

Throughout the paper that, for an algebra B, B_L will be the Lie algebra B with Lie bracket $[a, b] = ab - ba$ for all $a, b \in B$.

Let us review a definition of Poisson enveloping algebra (see [4, **3**]):

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Definition 1. *For a Poisson algebra A, a triple $(U(A), \alpha, \beta)$, where $U(A)$ is an algebra, $\alpha : A \longrightarrow U(A)$ is an algebra homomorphism and $\beta : A \longrightarrow U(A)_L$ is a Lie homomorphism such that*

$$\alpha(\{a, b\}) = [\beta(a), \alpha(b)], \quad \beta(ab) = \alpha(a)\beta(b) + \alpha(b)\beta(a)$$

for all $a, b \in A$, is called the Poisson enveloping algebra for A if $(U(A), \alpha, \beta)$ satisfies the following; if B is a k-algebra, γ is an algebra homomorphism from A into B and δ is a Lie homomorphism from $(A, \{\cdot, \cdot\})$ into B_L such that

$$\gamma(\{a, b\}) = [\delta(a), \gamma(b)], \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A$, then there exists a unique algebra homomorphism h from $U(A)$ into B such that $h\alpha = \gamma$ and $h\beta = \delta$.

$$\begin{tikzcd}
& U(A) \arrow[d, "h"] & \\
A \arrow[ur, "\alpha"] \arrow[dr, "\gamma"'] & & A \arrow[ul, "\beta"'] \arrow[dl, "\delta"] \\
& B &
\end{tikzcd}$$

For every Poisson algebra A over k, note that there exists a unique Poisson enveloping algebra $(U(A), \alpha, \beta)$ up to isomorphic, that $U(A)$ is generated by $\alpha(A)$ and $\beta(A)$ by [4, *proof of* **5**] *and that $\beta(1) = 0$.*

Definition 2. (see [3, 3.1.3]) *A Poisson algebra A is said to be a Poisson Hopf algebra if A is also a Hopf algebra $(A, \iota, \mu, \epsilon, \Delta, S)$ over k such that both structures are compatible in the sense that*

$$\Delta(\{a, b\}) = \{\Delta(a), \Delta(b)\}_{A \otimes A}$$

for all $a, b \in A$, where the Poisson bracket $\{\cdot, \cdot\}_{A \otimes A}$ on $A \otimes A$ is defined by

$$\{a \otimes a', b \otimes b'\}_{A \otimes A} = \{a, b\} \otimes a'b' + ab \otimes \{a', b'\}$$

for all $a, a', b, b' \in A$.

For example, every coordinate ring of Poisson Lie group is a Poisson Hopf algebra.

For Poisson algebras A and B, an algebra homomorphism $\phi : A \longrightarrow B$ is said to be a Poisson homomorphism (respectively, anti-homomorphism) if ϕ satisfies the rule

$$\phi(\{a, b\}) = \{\phi(a), \phi(b)\} \quad (\text{respectively, } \phi(\{a, b\}) = \{\phi(b), \phi(a)\})$$

for all $a, b \in A$.

Lemma 3. *If $(A, \iota, \mu, \epsilon, \Delta, S)$ is a Poisson Hopf algebra then the counit ϵ is a Poisson homomorphism and the antipode S is a Poisson anti-automorphism.*

Proof. [3, Remark 3.1.4] □

Lemma 4. *If γ and δ are k-linear maps from a Poisson algebra A into a k-algebra B such that*

$$\gamma(\{a,b\}) = [\delta(a),\gamma(b)], \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A$, then

$$\gamma(\{a,b\}) = [\gamma(a),\delta(b)], \quad \delta(ab) = \delta(a)\gamma(b) + \delta(b)\gamma(a).$$

Proof. Since

$$\begin{aligned}\gamma(\{a,b\}) + \delta(ab) &= \delta(a)\gamma(b) + \gamma(a)\delta(b)\\ \gamma(\{b,a\}) + \delta(ba) &= \delta(b)\gamma(a) + \gamma(b)\delta(a)\end{aligned}$$

we have

$$\begin{aligned}2\delta(ab) &= \delta(a)\gamma(b) + \delta(b)\gamma(a) + \gamma(a)\delta(b) + \gamma(b)\delta(a)\\ &= \delta(a)\gamma(b) + \delta(b)\gamma(a) + \delta(ab)\\ 2\gamma(\{a,b\}) &= \delta(a)\gamma(b) - \delta(b)\gamma(a) + \gamma(a)\delta(b) - \gamma(b)\delta(a)\\ &= \gamma(a)\delta(b) - \delta(b)\gamma(a) + \gamma(\{a,b\})\end{aligned}$$

by adding and subtracting the above two formulas. Hence we have the conclusion. □

Lemma 5. *Let $(U(A),\alpha,\beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then*
(i) *$\alpha\otimes\alpha : A\otimes A \longrightarrow U(A)\otimes U(A)$ is an algebra homomorphism.*
(ii) *$\alpha\otimes\beta+\beta\otimes\alpha : A\otimes A \longrightarrow (U(A)\otimes U(A))_L$ is a Lie homomorphism.*

Proof. (i) It is clear since α is an algebra homomorphism.
(ii) By Lemma 4, for $a, a', b, b' \in A$,

$$\begin{aligned}
&(\alpha\otimes\beta+\beta\otimes\alpha)(\{a\otimes a', b\otimes b'\})\\
&\qquad - [(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a'), (\alpha\otimes\beta+\beta\otimes\alpha)(b\otimes b')]\\
&= (\alpha\otimes\beta+\beta\otimes\alpha)(ab\otimes\{a',b'\} + \{a,b\}\otimes a'b')\\
&\qquad - [(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a'), (\alpha\otimes\beta+\beta\otimes\alpha)(b\otimes b')]\\
&= \alpha(ab)\otimes\beta(\{a',b'\}) + \alpha(\{a,b\})\otimes\beta(a'b') + \beta(ab)\otimes\alpha(\{a',b'\})\\
&\qquad + \beta(\{a,b\})\otimes\alpha(a'b') - [\alpha(a)\otimes\beta(a'), \alpha(b)\otimes\beta(b')]\\
&\qquad - [\alpha(a)\otimes\beta(a'), \beta(b)\otimes\alpha(b')] - [\beta(a)\otimes\alpha(a'), \alpha(b)\otimes\beta(b')]\\
&\qquad - [\beta(a)\otimes\alpha(a'), \beta(b)\otimes\alpha(b')]\\
&= \alpha(ab)\otimes[\beta(a'),\beta(b')] + [\beta(a),\alpha(b)]\otimes(\alpha(a')\beta(b') + \alpha(b')\beta(a'))\\
&\qquad + (\alpha(a)\beta(b) + \alpha(b)\beta(a))\otimes[\beta(a'),\alpha(b')] + [\beta(a),\beta(b)]\otimes\alpha(a'b')\\
&\qquad - \alpha(a)\alpha(b)\otimes[\beta(a'),\beta(b')] - \alpha(a)\beta(b)\otimes\beta(a')\alpha(b')\\
&\qquad + \beta(b)\alpha(a)\otimes\alpha(b')\beta(a') - \beta(a)\alpha(b)\otimes\alpha(a')\beta(b')\\
&\qquad + \alpha(b)\beta(a)\otimes\beta(b')\alpha(a') - [\beta(a),\beta(b)]\otimes\alpha(a')\alpha(b')\\
&= -\alpha(b)\beta(a)\otimes[\alpha(a'),\beta(b')] + [\beta(a),\alpha(b)]\otimes\alpha(b')\beta(a')\\
&\qquad - [\alpha(a),\beta(b)]\otimes\alpha(b')\beta(a') + \alpha(b)\beta(a)\otimes[\beta(a'),\alpha(b')]\\
&= -\alpha(b)\beta(a)\otimes\alpha(\{a',b'\}) + \alpha(\{a,b\})\otimes\alpha(b')\beta(a')\\
&\qquad - \alpha(\{a,b\})\otimes\alpha(b')\beta(a') + \alpha(b)\beta(a)\otimes\alpha(\{a',b'\})\\
&= 0.
\end{aligned}$$

Hence $\alpha\otimes\beta+\beta\otimes\alpha$ is a Lie homomorphism. $\square$

Lemma 6. *Let A and B be Poisson algebras and let C be an algebra. If $\phi: A\longrightarrow B$ is a Poisson homomorphism, $\alpha: B\longrightarrow C$ is an algebra homomorphism and $\beta: B\longrightarrow C_L$ is a Lie homomorphism such that*

$$\alpha(\{b_1,b_2\})=[\beta(b_1),\alpha(b_2)],\quad \beta(b_1b_2)=\alpha(b_1)\beta(b_2)+\alpha(b_2)\beta(b_1)$$

for all $b_1,b_2\in B$ then $\alpha\phi: A\longrightarrow C$ is an algebra homomorphism and $\beta\phi: A\longrightarrow C_L$ is a Lie homomorphism such that

$$\begin{aligned}\alpha\phi(\{a_1,a_2\})&=[\beta\phi(a_1),\alpha\phi(a_2)]\\ \beta\phi(a_1a_2)&=\alpha\phi(a_1)\beta\phi(a_2)+\alpha\phi(a_2)\beta\phi(a_1)\end{aligned}$$

for all $a_1,a_2\in A$.

Proof. Straightforward. $\square$

Lemma 7. *Let $(U(A),\alpha,\beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then $(U(A)\otimes U(A),\alpha\otimes\alpha,\alpha\otimes\beta+\beta\otimes\alpha)$ is the Poisson enveloping algebra for $A\otimes A$.*

Proof. It is straightforward to see that

$$\begin{aligned}(\alpha\otimes\alpha)(\{a\otimes a',b\otimes b'\})&=[(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a'),(\alpha\otimes\alpha)(b\otimes b')]\\ (\alpha\otimes\beta+\beta\otimes\alpha)((a\otimes a')(b\otimes b'))&=(\alpha\otimes\alpha)(a\otimes a')(\alpha\otimes\beta+\beta\otimes\alpha)(b\otimes b')\\ &\quad+(\alpha\otimes\alpha)(b\otimes b')(\alpha\otimes\beta+\beta\otimes\alpha)(a\otimes a').\end{aligned}$$

Let i_1 and i_2 be the Poisson homomorphisms from A into $A\otimes A$ defined by

$$\begin{aligned}i_1&: A\longrightarrow A\otimes A, & i_1(a)&=a\otimes 1\\ i_2&: A\longrightarrow A\otimes A, & i_2(a)&=1\otimes a\end{aligned}$$

for all $a\in A$. Given an algebra B, let μ_B be the multiplication map on B. If γ is an algebra homomorphism from $A\otimes A$ into B and δ is a Lie homomorphism from $A\otimes A$ into B_L such that

$$\begin{aligned}\gamma(\{a\otimes a',b\otimes b'\})&=[\delta(a\otimes a'),\gamma(b\otimes b')]\\ \delta((a\otimes a')(b\otimes b'))&=\gamma(a\otimes a')\delta(b\otimes b')+\gamma(b\otimes b')\delta(a\otimes a')\end{aligned}$$

for all $a,a',b,b'\in A$, then there exist algebra homomorphisms f,g from $U(A)$ into B such that $f\alpha=\gamma i_1, f\beta=\delta i_1, g\alpha=\gamma i_2, g\beta=\delta i_2$ by Lemma 6.

$$\begin{tikzcd}\mathrm{U(A)} \arrow[r, "f"] & \mathrm{B} \\ \mathrm{A} \arrow[u, "{\alpha,\beta}"] \arrow[r, "i_1"] & A\otimes A \arrow[u, "{\gamma,\delta}"']\end{tikzcd}\qquad\qquad \begin{tikzcd}\mathrm{U(A)} \arrow[r, "g"] & \mathrm{B} \\ \mathrm{A} \arrow[u, "{\alpha,\beta}"] \arrow[r, "i_2"] & A\otimes A \arrow[u, "{\gamma,\delta}"']\end{tikzcd}$$

Moreover we have $\delta i_1(a)\gamma i_2(a') = \gamma i_2(a')\delta i_1(a)$ for all $a, a' \in A$ since

$$\begin{aligned}[\delta i_1(a), \gamma i_2(a')] &= \gamma(\{a \otimes 1, 1 \otimes a'\}) \\ &= \gamma(a \otimes \{1, a'\} + \{a, 1\} \otimes a') \\ &= 0.\end{aligned}$$

Hence we have

$$\begin{aligned}\mu_B(f \otimes g)(\alpha \otimes \alpha)(a \otimes a') &= f\alpha(a)g\alpha(a') = \gamma i_1(a)\gamma i_2(a') \\ &= \gamma(i_1(a)i_2(a')) = \gamma(a \otimes a') \\ \mu_B(f \otimes g)(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes a') &= f\alpha(a)g\beta(a') + f\beta(a)g\alpha(a') \\ &= \gamma i_1(a)\delta i_2(a') + \delta i_1(a)\gamma i_2(a') \\ &= \gamma i_1(a)\delta i_2(a') + \gamma i_2(a')\delta i_1(a) \\ &= \delta(i_1(a)i_2(a')) \\ &= \delta(a \otimes a')\end{aligned}$$

for all $a, a' \in A$. Thus $\mu_B(f \otimes g)$ is an algebra homomorphism such that

$$\mu_B(f \otimes g)(\alpha \otimes \alpha) = \gamma, \quad \mu_B(f \otimes g)(\alpha \otimes \beta + \beta \otimes \alpha) = \delta.$$

If $h : U(A \otimes A) \longrightarrow B$ is an algebra homomorphism such that

$$h(\alpha \otimes \alpha) = \gamma, \quad h(\alpha \otimes \beta + \beta \otimes \alpha) = \delta$$

then

$$\begin{aligned}\mu_B(f \otimes g)(\alpha(a) \otimes 1) &= h(\alpha \otimes \alpha)(a \otimes 1) = h(\alpha(a) \otimes 1) \\ \mu_B(f \otimes g)(1 \otimes \alpha(a)) &= h(\alpha \otimes \alpha)(1 \otimes a) = h(1 \otimes \alpha(a)) \\ \mu_B(f \otimes g)(1 \otimes \beta(a)) &= h(\alpha \otimes \beta + \beta \otimes \alpha)(1 \otimes a) = h(1 \otimes \beta(a)) \\ \mu_B(f \otimes g)(\beta(a) \otimes 1) &= h(\alpha \otimes \beta + \beta \otimes \alpha)(a \otimes 1) = h(\beta(a) \otimes 1)\end{aligned}$$

for all $a \in A$, hence we have $\mu_B(f \otimes g) = h$ since $U(A)$ is generated by $\alpha(A)$ and $\beta(A)$. It completes the proof by Lemma 5. $\square$

Lemma 8. *Let $(U(A), \alpha_A, \beta_A)$ and $(U(B), \alpha_B, \beta_B)$ be Poisson enveloping algebras for Poisson algebras A and B respectively. If $\phi : A \longrightarrow B$ is a Poisson homomorphism then there exists a unique algebra homomorphism $U(\phi) : U(A) \longrightarrow U(B)$ such that $U(\phi)\alpha_A = \alpha_B\phi$ and $U(\phi)\beta_A = \beta_B\phi$.*

$$\begin{tikzcd} U(A) \arrow[r, "U(\phi)"] & U(B) \\ A \arrow[u, "{\alpha_A, \beta_A}"] \arrow[r, "\phi"] & B \arrow[u, "{\alpha_B, \beta_B}"'] \end{tikzcd}$$

Proof. It follows immediately from the definition for Poisson enveloping algebra and Lemma 6. $\square$

Let $A = (A, \cdot, \{\cdot,\cdot\})$ be a Poisson algebra. Define a k-bilinear map $\{\cdot,\cdot\}_1$ on A by

$$\{a,b\}_1 = \{b,a\}$$

for all $a, b \in A$. Then $A_1 = (A, \cdot, \{\cdot,\cdot\}_1)$ is a Poisson algebra. For an algebra B, we denote by $B^{op} = (B, \circ)$ the opposite algebra of B.

Proposition 9. *Let $(U(A), \alpha, \beta)$ be the Poisson enveloping algebra for a Poisson algebra A. Then $(U(A)^{op}, \alpha, \beta)$ is the Poisson enveloping algebra for A_1.*

Proof. Clearly, α is an algebra homomorphism from A_1 into $U(A)^{op}$ since A_1 is commutative and β is a Lie homomorphism from A_1 into $U(A)^{op}_L$. Moreover, by Lemma 4, we have

$$\alpha(\{a,b\}_1) = \alpha(\{b,a\}) = [\alpha(b), \beta(a)] = \beta(a)\circ\alpha(b) - \alpha(b)\circ\beta(a)$$
$$\beta(ab) = \beta(a)\alpha(b) + \beta(b)\alpha(a) = \alpha(a)\circ\beta(b) + \alpha(b)\circ\beta(a)$$

for all $a, b \in A_1$. If B is an algebra, $\gamma : A_1 \longrightarrow B$ is an algebra homomorphism and $\delta : A_1 \longrightarrow B_L$ is a Lie homomorphism such that

$$\gamma(\{a,b\}_1) = [\delta(a), \gamma(b)] \quad \text{and} \quad \delta(ab) = \gamma(a)\delta(b) + \gamma(b)\delta(a)$$

for all $a, b \in A_1$, then $\gamma : A \longrightarrow B^{op}$ is an algebra homomorphism and $\delta : A \longrightarrow B^{op}_L$ is a Lie homomorphism such that

$$\gamma(\{a,b\}) = \gamma(\{b,a\}_1) = [\gamma(b), \delta(a)] = \delta(a)\circ\gamma(b) - \gamma(b)\circ\delta(a)$$
$$\delta(ab) = \delta(a)\gamma(b) + \delta(b)\gamma(a) = \gamma(a)\circ\delta(b) + \gamma(b)\circ\delta(a)$$

for all $a, b \in A$ by Lemma 4. Hence there is a unique algebra homomorphism h from $U(A)$ into B^{op} such that $h\alpha = \gamma$ and $h\beta = \delta$ and so $h : U(A)^{op} \longrightarrow B$ is a unique algebra homomorphism such that $h\alpha = \gamma$ and $h\beta = \delta$. Thus $(U(A)^{op}, \alpha, \beta)$ is the Poisson enveloping algebra for A_1. $\square$

Theorem 10. *If $(A, \iota, \mu, \epsilon, \Delta, S)$ is a Poisson Hopf algebra then*

$$(U(A), \iota_{U(A)}, \mu_{U(A)}, U(\epsilon), U(\Delta), U(S))$$

is a Hopf algebra such that

$$U(\Delta)\alpha = (\alpha\otimes\alpha)\Delta \qquad U(\Delta)\beta = (\alpha\otimes\beta + \beta\otimes\alpha)\Delta$$
$$U(\epsilon)\alpha = \epsilon \qquad U(\epsilon)\beta = 0$$
$$U(S)\alpha = \alpha S \qquad U(S)\beta = \beta S.$$

Proof. Since Δ is a Poisson homomorphism and $(U(A)\otimes U(A), \alpha\otimes\alpha, \alpha\otimes\beta+\beta\otimes\alpha)$ is the Poisson enveloping algebra of $A\otimes A$ by Lemma 7, there exists an algebra homomorphism $U(\Delta): U(A)\longrightarrow U(A)\otimes U(A)$ such that

$$U(\Delta)\alpha = (\alpha\otimes\alpha)\Delta, \quad U(\Delta)\beta = (\alpha\otimes\beta+\beta\otimes\alpha)\Delta$$

by Lemma 8. Similarly, there exists an algebra homomorphism $U(\epsilon)$ from $U(A)$ into k such that $U(\epsilon)\alpha = \epsilon$, $U(\epsilon)\beta = 0$ since $(k, \mathrm{id}_k, 0)$ is the Poisson enveloping algebra of the field k with trivial Poisson bracket. Since the antipode S is a Poisson homomorphism from A into A_1 by Lemma 3, there is an algebra homomorphism $U(S): U(A)\longrightarrow U(A)^{op}$ such that $U(S)\alpha = \alpha S$ and $U(S)\beta = \beta S$ by Lemma 8 and Proposition 9. It is verified routinely that $(U(A), \iota_{U(A)}, \mu_{U(A)}, U(\epsilon), U(\Delta), U(S))$ is a Hopf algebra. $\square$

Example 11. Let L be a finite dimensional Lie algebra over k with Lie bracket $[\cdot,\cdot]$ and let $\mathcal{S}(L)$ be the symmetric algebra of L. Fix a k-basis $x_1,\dots,x_n$ of L. Note that $\mathcal{S}(L)$ is the commutative polynomial ring $k[x_1,\dots,x_n]$. Then, by [1, 2.8.7] or [2, Example 1], $\mathcal{S}(L)$ is a Poisson Hopf algebra with structure

$$\{a,b\} = [a,b],\ \Delta(a) = a\otimes 1 + 1\otimes a,\ \epsilon(a) = 0,\ S(a) = -a$$

for all $a, b\in L$. In fact, it is verified easily using the induction on degree of homogeneous elements of $\mathcal{S}(L)$ that

$$\begin{aligned}\Delta(\{x,y\}) &= \{\Delta(x),\Delta(y)\}\\ \epsilon(\{x,y\}) &= 0\\ S(\{x,y\}) &= \{S(y),S(x)\}\end{aligned}$$

for all $x,y\in\mathcal{S}(L)$. Observe that the Poisson enveloping algebra $U(\mathcal{S}(L)) = (U(\mathcal{S}(L)),\alpha,\beta)$ is the algebra generated by $x_1,\dots,x_n,y_1,\dots,y_n$ subject to the relation

$$x_ix_j = x_jx_i, \quad y_iy_j = y_jy_i + \psi([x_i,x_j]), \quad x_iy_j = y_jx_i + [x_i,x_j]$$

for all $i,j = 1,\dots,n$ and, α and β are given by $\alpha(x_i) = x_i$, $\beta(x_i) = y_i$, respectively, where $\psi: L\longrightarrow U(\mathcal{S}(L))$ is a k-linear map defined by $\psi(x_i) = y_i$ for all $i = 1,\dots,n$. By Theorem 10, the Poisson enveloping algebra $U(\mathcal{S}(L))$ is a Hopf algebra with Hopf structure

$$\begin{aligned}\Delta(x_i) &= x_i\otimes 1 + 1\otimes x_i & \Delta(y_i) &= y_i\otimes 1 + 1\otimes y_i\\ \epsilon(x_i) &= 0 & \epsilon(y_i) &= 0\\ S(x_i) &= -x_i & S(y_i) &= -y_i\end{aligned}$$

for all $i = 1,\dots,n$.

The Poisson enveloping algebra $U(\mathcal{S}(L))$ contains the universal enveloping algebra of L as a subalgebra. Let U be the subalgebra of $U(\mathcal{S}(L))$ generated by $y_1,\dots,y_n$ and let $j: L\longrightarrow U$ be a k-linear map defined by $j(x_i) = y_i$ for all $i = 1,\dots,n$. Then (U, j) is the universal enveloping algebra of L.

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