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The Largest Intersection Lattice of a Discriminantal Arrangement

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Abstract. We prove a conjecture of Bayer and Brandt [*J. Alg. Combin.* **6** (1997), 229–246] about the "largest" intersection lattice of a discriminantal arrangement based on an essential arrangement of n linear hyperplanes in $\mathbb{R}^k$. An important ingredient in the proof is Crapo's characterization of the matroid of circuits of the configuration of n generic points in $\mathbb{R}^k$.

1. Introduction

Let $\mathcal{A}$ be an arrangement of n linear hyperplanes in $\mathbb{R}^k$ with normal vectors α_i for $1 \le i \le n$. Assume that the vectors α_i span $\mathbb{R}^k$, so that $\mathcal{A}$ is *essential.* The *discriminantal arrangement* $\mathcal{B}(\mathcal{A})$ based on $\mathcal{A}$ (see [11], [1]) is the arrangement of linear hyperplanes in $\mathbb{R}^n$ with normal vectors the distinct, nonzero vectors of the form

$$\alpha_S = \sum_{i=1}^{k+1} (-1)^i \det\left(\alpha_{s_1}, \ldots, \alpha_{s_{i-1}}, \alpha_{s_{i+1}}, \ldots, \alpha_{s_{k+1}}\right) e_{s_i}, \tag{1}$$

where $S = \{s_1 < s_2 < \cdots < s_{k+1}\}$ ranges over all subsets of $[n] := \{1, 2, \ldots, n\}$ with $k+1$ elements and the e_j are the standard coordinate vectors in $\mathbb{R}^n$.

We are primarily concerned with a conjecture of Bayer and Brandt [1] about the *intersection lattice* [13, §2.1] $L(\mathcal{B}(\mathcal{A}))$ of $\mathcal{B}(\mathcal{A})$, the main combinatorial invariant of $\mathcal{B}(\mathcal{A})$. Bayer

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and Brandt call $\mathcal{A}$ *very generic* if for all r, the number of elements of $L(\mathcal{B}(\mathcal{A}))$ of rank r is the largest possible for a discriminantal arrangement based on an essential arrangement of n linear hyperplanes in $\mathbb{R}^k$. They denote by $P(n,k)$ the collection of all sets of the form $\{S_1, S_2, \ldots, S_m\}$, where the S_i are subsets of $[n]$, each of cardinality at least $k+1$, such that

$$|\bigcup_{i\in I} S_i| > k + \sum_{i\in I} (|S_i| - k)$$

for all $I \subseteq [m]$ with $|I| \geq 2$. They partially order $P(n,k)$ by letting $\mathcal{S} \preceq \mathcal{T}$ if every subset in $\mathcal{S}$ is contained in some subset in $\mathcal{T}$. Note that $P(n,1)$ is isomorphic to the lattice of partitions of $[n]$. The following appears as Conjecture 4.3 in [1].

Conjecture 1.1. (Bayer and Brandt [1]) *Let $n \geq k+1 \geq 2$. There exist arrangements of n hyperplanes in $\mathbb{R}^k$ which are very generic. Moreover, the intersection lattice of the discriminantal arrangement based on any very generic arrangement of n hyperplanes in $\mathbb{R}^k$ is isomorphic to $P(n,k)$.*

The discriminantal arrangement was originally defined when the hyperplanes of $\mathcal{A}$ are in general position, meaning that any k of the vectors α_i are linearly independent, by Manin and Schechtman [11], who initiated the study of its complement. Manin and Schechtman remarked [11, p. 292] that although $\mathcal{B}(\mathcal{A})$ depends on the original arrangement $\mathcal{A}$ (see also Falk [9]), its intersection lattice depends only on n and k if $\mathcal{A}$ lies in a Zariski open subset of the space of all arrangements of n linear hyperplanes in general position in $\mathbb{R}^k$. They gave a description of this lattice, which we denote by $L(n,k)$, for $k+1 \leq n \leq k+3$ ([11, p. 292], see also [13, p. 205]) and implicitly suggested the problem of describing $L(n,k)$ for other values of n.

From a different perspective, unkown in the literature of discriminantal arrangements, the matroid of circuits of the configuration of n points $\alpha_1, \alpha_2, \ldots, \alpha_n$ in $\mathbb{R}^k$, realized by the vectors α_S, has been introduced by Crapo [4]. Crapo characterized this matroid $M(n,k)$ when the coordinates of the vectors α_i are generic indeterminates [5] as the *Dilworth completion* $D_k(B_n)$ of the kth lower truncation of the Boolean algebra of rank n (see e.g. Crapo and Rota [6, §7], Mason [12] or Brylawski [3, §7]). The lattice $L(n,k)$ is the geometric lattice of flats of $M(n,k)$. For recent related work, see Crapo and Rota [7].

In the present paper we first define precisely the Zariski open set $\mathcal{O}(n,k)$ that the hypothesis of Manin and Schechtman on $\mathcal{A}$ refers to, in order to clarify this point (see [9, p. 1222]). We refer to an arrangement $\mathcal{A}$ in $\mathcal{O}(n,k)$ as *sufficiently general.* We use ideas from matroid theory [10] to show (Theorem 2.3) that any arrangement $\mathcal{A}$ in $\mathcal{O}(n,k)$ is very generic and moreover, that it maximizes the f-vector of $\mathcal{B}(\mathcal{A})$ or, equivalently, the f-vector of the corresponding fiber zonotope [2, §4]. It follows that very generic arrangements exist and that the lattice which Conjecture 1.1 refers to coincides with $L(n,k)$. Finally we show (Theorem 3.2) that Crapo's characterization [5, §6] of $M(n,k)$ implies that its lattice of flats $L(n,k)$ is isomorphic to $P(n,k)$. Equivalently, the lattice of flats of the Dilworth completion $D_k(B_n)$ is isomorphic to $P(n,k)$. This has been observed earlier for $k=1$ ([6, Prop. 7.9], [12, §2.5]) and completes the proof of Conjecture 1.1.

2. Sufficiently general arrangements

Let $\mathcal{A}$ be an essential arrangement of n linear hyperplanes in $\mathbb{R}^k$, as in the beginning of the introduction, with normal vectors $\alpha_i = (\alpha_{ij})_{j=1}^k$ for $1 \leq i \leq n$. For each set $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ of $(k+1)$-subsets of $[n]$ with $m \leq n$, we denote by $A_{\mathcal{S}}$ the $m \times n$ matrix whose (r, j) entry is the jth coordinate of the vector α_{S_r} in $\mathbb{R}^n$. Let $p_{\mathcal{S}}$ be the sum of the squares of the $m \times m$ minors of the matrix $A_{\mathcal{S}}$, considered as a polynomial in the indeterminates α_{ij}. Thus $p_{\mathcal{S}}$ is an element of the ring $\mathbb{R}[x_{ij}]$ and $p_{\mathcal{S}}(\alpha_{ij}) = 0$ if and only if the vectors $\alpha_{S_1}, \alpha_{S_2}, \ldots, \alpha_{S_m}$ are linearly dependent.

Definition 2.1. *We call $\mathcal{A}$ sufficiently general if $p_{\mathcal{S}}(\alpha_{ij}) \neq 0$ for all $\mathcal{S}$ for which the polynomial $p_{\mathcal{S}}$ is not identically zero. We denote by $\mathcal{O}(n, k)$ the space of sufficiently general arrangements.*

The set $\mathcal{O}(n, k)$ is a nonempty Zariski open subset of the space of all arrangements of n linear hyperplanes in general position in $\mathbb{R}^k$. Apparently, and in view of the following easy proposition, it is the Zariski open set that Manin and Schechtman refer to.

Proposition 2.2. *If $\mathcal{A}$ is restricted in the set $\mathcal{O}(n, k)$ then the matroid realized by the vectors α_S, and hence the intersection lattice $L(n, k)$ as well as the f-vector of $\mathcal{B}(\mathcal{A})$, depend only on n and k.*

Proof. Indeed, a set $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ indexes an independent subset of the set of vectors $\{\alpha_S\}$ if and only if the polynomial $p_{\mathcal{S}}$ is not identically zero. □

We now use the idea of *weak maps* of matroids [10] to show that any arrangement in $\mathcal{O}(n, k)$ is very generic, in the sense of Bayer and Brandt. A different application of the same technique was recently given by Edelman [8].

Theorem 2.3. *If $\mathcal{A}$ is in $\mathcal{O}(n, k)$ then $\mathcal{B}(\mathcal{A})$ has maximum f-vector and intersection lattice with all rank sets of maximum size among all discriminantal arrangements based on an essential arrangement of n linear hyperplanes in $\mathbb{R}^k$.*

Proof. Let $\mathcal{A}'$ be any essential arrangement of n linear hyperplanes in $\mathbb{R}^k$ with corresponding data α'_i, α'_{ij} and α'_S. The map $\alpha_S \to \alpha'_S$ defines a weak map [10] of the matroids M and M', realized by the sets of vectors $\{\alpha_S\}$ and $\{\alpha'_S\}$ respectively. Indeed, if the $\{\alpha'_S\}$ for $S \in \mathcal{S}$ are linearly independent then $p_{\mathcal{S}}(\alpha'_{ij}) \neq 0$, so $p_{\mathcal{S}}$ is not identically zero and hence the $\{\alpha_S\}$ for $S \in \mathcal{S}$ are also linearly independent. The result follows from the fact that the Whitney numbers of both kinds of M' are bounded above by those of M whenever there is a weak map of matroids $M \to M'$ (see Proposition 9.3.3 and Corollary 9.3.7 in [10]). □

3. The intersection lattice

We first recall some useful notation from [7]. A set $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ of subsets of $[n]$ is an *antichain* if none of the subsets contains another. For such an antichain $\mathcal{S}$ we let

$$\Delta(\mathcal{S}) = \nu\,(\bigcup_{S \in \mathcal{S}} S) - \sum_{S \in \mathcal{S}} \nu\,(S),$$

where $\nu(S) = \max(0, |S| - k)$. Thus $\mathcal{S} \in P(n,k)$ if each S_i has cardinality at least $k+1$ and $\Delta(\mathcal{F}) > 0$ for all $\mathcal{F} \subseteq \mathcal{S}$ with $|\mathcal{F}| \geq 2$. If $\mathcal{T} = \{T_1, T_2, \ldots, T_p\}$ is another antichain, we let $\mathcal{S} \preceq \mathcal{T}$ if for each i there exists a j such that $S_i \subseteq T_j$.

We assume that $\mathcal{A}$ is in $\mathcal{O}(n,k)$ and think of $L(n,k)$ as the geometric lattice spanned by the vectors α_S. The matroid $M(n,k)$ realized by these vectors, to which we have referred in Proposition 2.2, has been characterized by Crapo [5, §6] and was shown to be isomorphic to the Dilworth completion $D_k(B_n)$ ([6, §7], [12]) of the kth lower truncation of the Boolean algebra of rank n. The Dilworth completion $D_k(B_n)$ is a matroid on the set of all $(k+1)$-subsets of $[n]$ which can be described explicitly in terms of its bases, independent sets and circuits. We keep our discussion as self-contained as possible. The following result is equivalent to Theorem 2 in [5, §6].

Theorem 3.1. (Crapo [5]) *Let $S_1, S_2, \ldots, S_m$ be $(k+1)$-subsets of $[n]$. The vectors α_{S_1}, $\alpha_{S_2}, \ldots, \alpha_{S_m}$ are linearly independent if and only if*

$$|\bigcup_{i \in I} S_i| \geq k + |I| \tag{2}$$

for all $I \subseteq [m]$ or, in other words, if $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ satisfies $\Delta(\mathcal{F}) \geq 0$ for all $\mathcal{F} \subseteq \mathcal{S}$.

Proof. The elegant argument in the proof of Theorem 2 in [5, §6] shows that $\alpha_{\mathcal{S}} := \{\alpha_{S_1}, \alpha_{S_2}, \ldots, \alpha_{S_m}\}$ is a basis if and only if $\Delta(\mathcal{F}) \geq 0$ for all $\mathcal{F} \subseteq \mathcal{S}$ and $m = n - k$. It suffices to show that if $\mathcal{S}$ satisfies (2) then $\alpha_{\mathcal{S}}$ is contained in a basis. This follows by an application of Hall's marriage theorem to the sets $S_i - [k]$. □

In the remaining of this section we show that this characterization of $M(n,k)$ implies that its lattice of flats $L(n,k)$ is isomorphic to the poset $P(n,k)$, defined in the introduction. For an antichain $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ of subsets of $[n]$ we denote by $V_{\mathcal{S}}$ the linear span of the vectors α_S for all $(k+1)$-subsets S of $[n]$ such that $S \subseteq S_i$ for some $1 \leq i \leq m$. If $\mathcal{S} = \{S\}$, where S has any cardinality, we write V_S instead of $V_{\mathcal{S}}$. Note that $\dim V_S = \nu(S)$, which clearly implies the "only if" direction of Theorem 3.1. We let $\varphi(\mathcal{S}) = V_{\mathcal{S}}$ if $\mathcal{S} \in P(n,k)$.

Theorem 3.2. *The map $\varphi : P(n,k) \to L(n,k)$ is an isomorphism of posets.*

We give the proof after having established a few lemmas.

Lemma 3.3. *Let $\mathcal{S}$ be an antichain with the properties $|S| \geq k+1$ for all $S \in \mathcal{S}$, $\Delta(\mathcal{F}) \geq 0$ for all $\mathcal{F} \subseteq \mathcal{S}$ and $V_{\mathcal{S}} = \bigoplus_{S \in \mathcal{S}} V_S$. If $\mathcal{S}$ is not in $P(n,k)$ then there exists an antichain $\mathcal{S}'$ with the same three properties as $\mathcal{S}$ and such that $V_{\mathcal{S}'} = V_{\mathcal{S}}$, $|\mathcal{S}'| < |\mathcal{S}|$ and $\mathcal{S} \prec \mathcal{S}'$.*

Proof. By assumption, $\Delta(\mathcal{F}) = 0$ for some $\mathcal{F} \subseteq \mathcal{S}$ with at least two elements. Let U be the union of the elements of $\mathcal{F}$. Since $V_F \subseteq V_U$ for all $F \in \mathcal{F}$, the sum of the V_F is direct and their dimensions $\nu(F)$ sum to the dimension $\nu(U)$ of V_U, we have

$$V_U = \bigoplus_{F \in \mathcal{F}} V_F.$$

Replace the subsets $F \in \mathcal{F}$ in $\mathcal{S}$ with their union U to get the desired antichain $\mathcal{S}'$. The property $\Delta(\mathcal{E}) \geq 0$ for all $\mathcal{E} \subseteq \mathcal{S}'$ follows from the corresponding property of $\mathcal{S}$ and the fact that $\nu(U) = \sum_{F \in \mathcal{F}} \nu(F)$. □

Lemma 3.4. *If $\mathcal{S}$ and $\mathcal{T}$ are any antichains with $\mathcal{S} \preceq \mathcal{T}$ and $\Delta(\mathcal{F}) \geq 0$ for all $\mathcal{F} \subseteq \mathcal{S}$, then*

$$\sum_{S \in \mathcal{S}} \nu(S) \leq \sum_{T \in \mathcal{T}} \nu(T).$$

Proof. We induct on the cardinality of $\mathcal{S}$. Choose a set $T \in \mathcal{T}$ so that the subfamily $\mathcal{F} = \{F \in \mathcal{S} : F \subseteq T\}$ is nonempty. Then

$$\sum_{S \in \mathcal{S}} \nu(S) = \sum_{F \in \mathcal{F}} \nu(F) + \sum_{S \in \mathcal{S} - \mathcal{F}} \nu(S) \leq \nu(\bigcup_{F \in \mathcal{F}} F) + \sum_{S \in \mathcal{S} - \mathcal{F}} \nu(S) \leq$$

$$\leq \nu(T) + \sum_{S \in \mathcal{S} - \mathcal{F}} \nu(S).$$

Since $\mathcal{S} - \mathcal{F} \preceq \mathcal{T} - \{T\}$, induction completes the proof. □

Lemma 3.5. *If $\mathcal{S} \in P(n,k)$, T is a subset of $[n]$ with $|T| \geq k+1$ and $\{R\} \preceq \mathcal{S}$ for all $R \subseteq T$ with $|R| = k+1$, then $\{T\} \preceq \mathcal{S}$.*

Proof. This follows from the definition of $P(n,k)$ by an easy induction. □

We now prove Theorem 3.2 and thus complete the proof of Conjecture 1.1.

Proof of Theorem 3.2. The map φ is clearly order preserving. We first show that it is surjective. Let V be a linear space in $L(n,k)$. Choose a basis $\{\alpha_{S_1}, \alpha_{S_2}, \ldots, \alpha_{S_m}\}$ of V, where the S_i are $(k+1)$-subsets of $[n]$ and let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$, so that $V = V_{\mathcal{S}}$. If $\mathcal{S} \in P(n,k)$ then $\varphi(\mathcal{S}) = V$. If not, then Lemma 3.3 applies and produces a sequence of antichains $\mathcal{S} \prec \mathcal{S}_1 \prec \mathcal{S}_2 \prec \cdots$ with $V = V_{\mathcal{S}_1} = V_{\mathcal{S}_2} = \cdots$. Since this procedure reduces m at each step, it terminates. The last antichain $\mathcal{T}$ in the sequence is in $P(n,k)$ and satisfies $\varphi(\mathcal{T}) = V$.

Next we show that φ is injective. Suppose on the contrary that $\varphi(\mathcal{S}) = \varphi(\mathcal{T})$ for two distinct families $\mathcal{S}, \mathcal{T} \in P(n,k)$. We may assume that $\mathcal{T} \preceq \mathcal{S}$ is invalid. By Lemma 3.5 there exists a $(k+1)$-set R contained in some subset in $\mathcal{T}$ but not contained in any $S \in \mathcal{S}$. Since $\alpha_R \in V_{\mathcal{S}}$, we can choose a *minimal* family $\mathcal{F} \preceq \mathcal{S}$ of $(k+1)$-subsets such that $\{\alpha_F : F \in \mathcal{F}\}$ is independent and α_R lies in its span. Theorem 3.1 and the minimality of $\mathcal{F}$ imply that

$$\nu\left((\bigcup_{F \in \mathcal{F}} F) \cup R \right) < |\mathcal{F}| + 1.$$

On the other hand

$$\nu(\bigcup_{F \in \mathcal{F}} F) \geq |\mathcal{F}|,$$

by independence of the α_F. Hence $R \subseteq \bigcup_{F \in \mathcal{F}} F$ and

$$\nu(\bigcup_{F \in \mathcal{F}} F) = |\mathcal{F}|.$$

Let $\mathcal{E}$ be the subfamily consisting of the subsets $E \in \mathcal{S}$ which contain some $F \in \mathcal{F}$ and $\mathcal{E}'$ be the family obtained from $\mathcal{E}$ by intersecting all sets in $\mathcal{E}$ with $\bigcup_{F\in\mathcal{F}} F$, so that $\mathcal{F} \preceq \mathcal{E}'$. We have

$$\nu\,(\bigcup_{E'\in\mathcal{E}'} E') = \nu\,(\bigcup_{F\in\mathcal{F}} F) = \sum_{F\in\mathcal{F}} \nu\,(F) \leq \sum_{E'\in\mathcal{E}'} \nu\,(E'), \tag{3}$$

where the inequality follows from Lemma 3.4. It follows that

$$\nu\,(\bigcup_{E\in\mathcal{E}} E) \leq \sum_{E\in\mathcal{E}} \nu\,(E), \tag{4}$$

since adding elements to each E' preserves the inequality (3). By the choice of R, $\mathcal{E}$ has cardinality at least 2. Hence (4) contradicts the fact that $\mathcal{S} \in P(n,k)$.

Finally we show that the inverse of φ is order preserving. Let $\mathcal{S}, \mathcal{T} \in P(n,k)$ be such that $\varphi(\mathcal{S}) = V_{\mathcal{S}} \subseteq V_{\mathcal{T}} = \varphi(\mathcal{T})$. Choose a basis $\{\alpha_S : S \in \mathcal{S}_0\}$ for $V_{\mathcal{S}}$ and complete it to a basis $\{\alpha_S : S \in \mathcal{T}_0\}$ of $V_{\mathcal{T}}$, where $\mathcal{S}_0 \subseteq \mathcal{T}_0$ are families of $(k+1)$-subsets of $[n]$. Apply Lemma 3.3 successively to $\mathcal{S}_0$ to find an $\mathcal{S}_1 \in P(n,k)$ with $\mathcal{S}_0 \preceq \mathcal{S}_1$ and $\varphi(\mathcal{S}_1) = V_{\mathcal{S}} = \varphi(\mathcal{S})$. Then apply the same lemma to $\mathcal{S}_1 \cup (\mathcal{T}_0 - \mathcal{S}_0)$ to find a $\mathcal{T}_1 \in P(n,k)$ with $\mathcal{S}_1 \preceq \mathcal{T}_1$ and $\varphi(\mathcal{T}_1) = V_{\mathcal{T}} = \varphi(\mathcal{T})$. By injectivity of φ we get $\mathcal{S} = \mathcal{S}_1$ and $\mathcal{T} = \mathcal{T}_1$ so $\mathcal{S} \preceq \mathcal{T}$, as desired. □

The following is a corollary of Theorem 3.2 and the fact that the operation $\mathcal{S} \mapsto \mathcal{S}'$ of Lemma 3.3 preserves the quantity $\sum_{S\in\mathcal{S}} \nu\,(S)$.

Corollary 3.6. *The poset $P(n,k)$ is a geometric lattice with rank function*

$$r(\mathcal{S}) = \sum_{S\in\mathcal{S}} \nu\,(S).$$

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References

[1] Bayer, M.; Brandt, K. A.: *Discriminantal arrangements, fiber polytopes and formality.* J. Algebraic Combin. **6** (1997), 229–246.

[2] Billera. L. J.; Sturmfels, B.: *Fiber polytopes.* Ann. of Math. **135** (1992), 527–549.

[3] Brylawski. T.: *Constructions.* Chapter 7 in: Theory of Matroids (N. White ed.), 127–223, Encyclopedia Math. Appl. **26**, Cambridge University Press, 1986.

[4] Crapo, H.: *Concurrence geometries.* Adv. Math. **54** (1984), 278–301.

[5] Crapo, H.: *The combinatorial theory of structures.* In: Matroid theory (A. Recski and L. Lovász eds.), Colloq. Math. Soc. János Bolyai **40**, 107–213, North-Holland, Amsterdam-New York 1985.

[6] Crapo, H.; Rota, G.-C.: *On the Foundations of Combinatorial Theory: Combinatorial Geometries.* Preliminary edition, M.I.T. Press, Cambridge, MA, 1970.

[7] Crapo, H.; Rota, G.-C.: *The resolving bracket.* In: Invariant methods in discrete and computational geometry (N. White ed.), 197–222, Kluwer Academic Publishers, Dordrecht 1995.

[8] Edelman, P. H.: *Ordering points by linear functionals.* Preprint dated March 11, 1998.

[9] Falk, M.: *A note on discriminantal arrangements.* Proc. Amer. Math. Soc. **122** (1994), 1221–1227.

[10] Kung, J. P. S.; Nguyen, H. Q.: *Weak maps.* Chapter 9 in: Theory of Matroids (N. White ed.), 254–271, Encyclopedia Math. Appl. **26**, Cambridge University Press 1986.

[11] Manin, Y. I.; Schechtman. V. V.: *Arrangements of hyperplanes, higher braid groups and higher Bruhat orders.* In: Algebraic Number Theory – in honor of K. Iwasawa, 289–308, Adv. Stud. Pure Math. **17**, 1989.

[12] Mason, J. H.: *Matroids as the study of geometrical configurations.* In: Higher combinatorics (M. Aigner ed.), 133–176, Reidel, Dordrecht–Boston, MA, 1977.

[13] Orlik, P.; Terao, H.: *Arrangements of Hyperplanes.* Grundlehren Math. Wiss. **300**, Springer-Verlag, New York, NY, 1992.

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