SOME PROBLEMS ON COMBINATIONAL LOGICAL CIRCUITS

by
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Abstract: In this paper we study some constructions of big combinational logical circuits from smaller combinational logical circuits. We also present the set-theoretical Yang-Baxter equation, and show that the problems presented in this paper are related to it.

1. INTRODUCTION

The present paper represents a connection between the two scientific directions
treated by this journal: mathematics and informatics.

This paper is related to a previous paper on combinational logical circuits which appeared in this journal (see [1]). In that paper, the authors presented an application of feed-forward neural networks: the simulation of combinational logical circuits, CLC's for short. For more details on neural networks we refer to [2] and [3].

We will consider some different constructions in this paper. Starting with small combinational logical circuits, we build bigger combinational logical circuits.

Our paper is organized as follows. Section 2 is an informal introduction to combinational logical circuits. Our material might be sufficient for readers with background in the area, but for those who need a review on combinational logical circuits we recommend [1]. We also define when two CLC's are equivalent. In section 3, we give a set of problems. For some of them we give solutions. In section 4, we present the set-theoretical Yang-Baxter equation. We show that the problems 2 and 3 are a particular case of this famous equation. On the other hand, they highlight the signification of this equation.

2. PRELIMINARIES

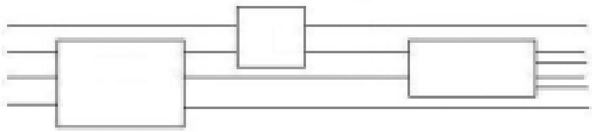
A combinational logical circuit (CLC) is an electronic circuit with n inputs, and m outputs, for which the outputs could be expresed according to the inputs.

We propose the following notation for this combinational logical circuit: CLC[n,m].

The following are notations from [1]:

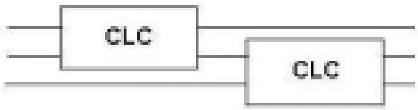
X - the set of input variables,
Z - the set of output variables
F:X→Z - the input-output function.

Starting with small combinational logical circuits, we can build a bigger CLC:



In this paper we start the study of this type of constructions. Since the general case is too complex, we will consider just constructions of combinational logical circuits with 3 inputs and 3 outputs, denoted CLC[3,3], from combinational logical circuits with 2 inputs and 2 outputs, CLC[2,2]. For example, we can use two identical CLC[2,2]

to construct the following CLC[3,3]:



Of course, there are other ways to construct a CLC[3,3] !

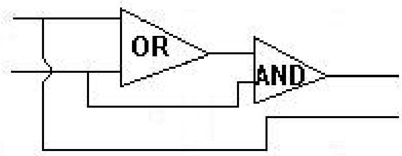
So, we give the following definition.

Definition. Two combinational logical circuits with the input-output functions
F:X→Z,
G:X→Z
are called *equivalent* if:

F(0,0, …,0,0) = G(0,0,…,0,0)
F(0,0, …,0,1) = G(0,0,…,0,1)
…………….
F(1,1, …,1,0) = G(1,1,…,1,0)
F(1,1, …,1,1) = G(1,1,…,1,1)

Example of equivalent CLC's. Let us consider the following two CLC[2,2] :

i)

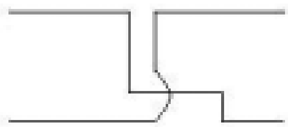


the input-output function:

F:X={(0,0),(0,1),(1,0),(1,1)}→**Z**={(0,0),(0,1),(1,0),(1,1)} **,**

F (X,Y) = ($(X\vee Y)\wedge Y$, X)

ii)



the input-output function:

G:X={(0,0),(0,1),(1,0),(1,1)}→**Z**={(0,0),(0,1),(1,0),(1,1)} ,

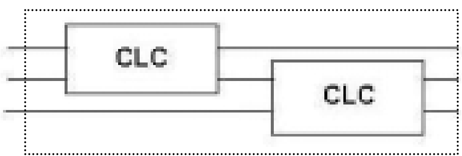
G(X,Y) = (Y,X)

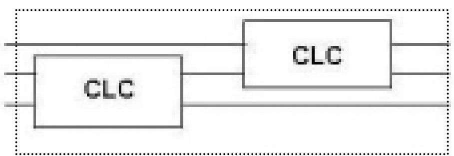
Proof. We just need to check the definition:

$$F(0,0) = ((0\vee 0)\wedge 0,\ 0) = (0\wedge 0,\ 0) = (0,0) = G(0,0)$$
$$F(0,1) = ((0\vee 1)\wedge 1,\ 0) = (1\wedge 0,\ 0) = (1,0) = G(0,1)$$
$$F(1,0) = ((1\vee 0)\wedge 0,\ 1) = (1\wedge 0,\ 1) = (0,1) = G(1,0)$$
$$F(1,1) = ((1\vee 1)\wedge 1,\ 1) = (1\wedge 1,\ 1) = (1,1) = G(1,1)$$

3. SOME NEW PROBLEMS

Problem 1. Starting with two identical CLC[2,2] we can construct a CLC[3,3] in the following two ways:



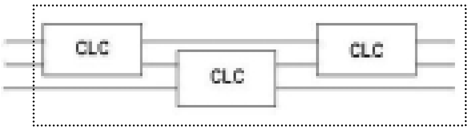


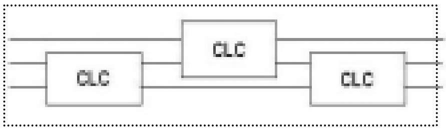
Find a CLC[2,2] such that these two bigger combinational logical circuits are *equivalent.*

Solution. Hint: Construct a CLC[2,2] for the following input-output function

G:X→Z , **G**(0,0) = (1,1) **G**(1,0) = (0,1) **G**(0,1) = (1,0) **G**(1,1) = (0,0).

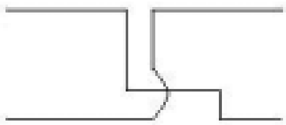
Problem 2. Starting with three identical CLC[2,2] we can construct a CLC[3,3] in the following two ways:





Find a CLC[2,2] such that these two bigger combinational logical circuits are *equivalent.*

Solution. This CLC[2,2] might be the following:



The input-output function is **G:X→Z** , **G**(X,Y) = (Y,X).
(*The reader should check the details*.)

Problem 3. Find all solutions for Problem 2.

Solution. *The solution will be treated in another paper.*

4. THE SET-THEORETICAL YANG-BAXTER EQUATION

The Yang-Baxter equation plays an important role in theoretical physics, knot theory and quantum groups (see [4], [6]). Many papers in the literature are devoted to finding solutions for it.

Finding all solutions for this equation (the classification of solutions) is an even harder problem. A complete classification was obtained only in dimension 2, using computer calculations.

We present bellow the set-theoretical version of the Yang-Baxter equation (see [5]).

Let S be a set and $R: S\times S \to S\times S$, $R(u,v)=(u',v')$ be a function.

We use the following notations:

$$R\times Id : S\times S\times S\rightarrow S\times S\times S, \quad R\times Id(u,v,w) = (u',v',w)$$
$$Id\times R : S\times S\times S\rightarrow S\times S\times S, \quad Id\times R(u,v,w) = (u,v',w')$$

The following is the set-theoretical Yang-Baxter equation:

$$(R\times Id)o(Id\times R)o(R\times Id) = (Id\times R)o(R\times Id)o(Id\times R)$$

If the cardinality of S is 2 solving the set-theoretical Yang-Baxter equation is the same as solving ***Problems 2*** and ***3.*** The computations are difficult, even in this particular case; so, we need to use computer calculations. This gives a grasp about the difficulty of the original problem.

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