GAMS LANGUAGE IN DESCRIPTION OF METHOD OF MINIMUM TOTAL VARIATION

by
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Abstract. The article show how can be used GAMS (General Algebric Modeling System) in description of the method of minimum total variation. GAMS is a language and can be used so solve any problem of linear programation or nonlinear programation. In this article Gams is used to find the estimations of parameters which are in linear or nonlinear relations using the method of minimum total variation.

We consider an econometric model with n equations which can be show as follows:

$$(*) \qquad y = Y + e \ , \ Y = F(X, \theta)$$

where, F is a real function , $Y^T = (Y_1, Y_2, ..., Y_n)$ are the vector of dependent variables, variables whose values can be obtain statistical direct and $\theta^T = (\theta_1, \theta_2, ..., \theta_m)$ is the parameters vector, parameters which must be estimated. We have the errors vector $e^T = (e_1, e_2, ..., e_n)$ and the matrices of independent variables,

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1m} \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nm} \end{pmatrix}$$

variables which are known or can be known after a statistical observation.
We can have some restriction of the parameters. In this situations we have the next equations attached to the model (*):

$$H_1(\theta_1, \theta_2, ..., \theta_m) = 0$$
$$H_2(\theta_1, \theta_2, ..., \theta_m) = 0 \qquad (1)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$H_r(\theta_1, \theta_2, ..., \theta_m) = 0, \text{ with } r<m.$$

At these equations we consider the random estimation of vector θ components and in the same time the system of the experimental equations:

$$y=Y+e \ , \ Y=\theta .$$

The errors e_i , i=1,2,…,n can be independent or dependent one other. If the errors are independent then $E(e_i e_j) = 0$, if $i \neq j$.

If we presume that only the first l with l<m parameters from equations (1) can be observe statistical direct and that this is the only sources of information we have a problem of estimation where the parameters are dependents one another.

This new problem mathematical representation will be:

$$H_1(\theta_1,\theta_2,...,\theta_l,\theta_{l+1},...,\theta_m)=0$$
$$H_2(\theta_1,\theta_2,...,\theta_l,\theta_{l+1},...,\theta_m)=0 \qquad (2)$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$H_r(\theta_1,\theta_2,...,\theta_l,\theta_{l+1},...,\theta_m)=0$$

The system of experimental equations of the equations (2) can be written:

$$y_{n1}=Y_{l1}+e_{l1}\ ,\quad Y_{l1}=\theta_{l1}$$

We consider the parameters unknown from the equations (1). If consider them free parameters we can estimate using an estimation method the estimations of parameters note:

$$\hat{\theta}_1^{(0)},\ \hat{\theta}_2^{(0)},\ \ldots,\hat{\theta}_m^{(0)}. \qquad (3)$$

If this estimations verify the equations (1), thing that is little probable, then the problem can be consider solve.

We presume that the estimations (3) didn't verify the equations (1). We can use than this results like source of information for the parameters $\theta_1,\theta_2,...,\theta_m$. Can be found limits for the parameters as follows:

$$\hat{\theta}_j^{(0)}-\Delta\hat{\theta}_j^{(0)}\le\theta_j\le\hat{\theta}_j^{(0)}+\Delta\hat{\theta}_j^{(0)} \quad ,\ j=1,2,\ldots,m.$$

For estimation of the parameters $\theta_1,\theta_2,...,\theta_m$ we use two groups of restrictions:

$$H_k(\theta_1,\theta_2,...,\theta_m)=0 \quad ,\ k=1,2,\ldots,r.$$
$$\hat{\theta}_j^{(0)}-\Delta\hat{\theta}_j^{(0)}\le\theta_j\le\hat{\theta}_j^{(0)}+\Delta\hat{\theta}_j^{(0)}\ ,\ j=1,2,\ldots,m. \qquad (4)$$

Let it be the estimations $\hat{\theta}_1,\hat{\theta}_2,...,\hat{\theta}_m$ verifying the conditions (4) and in the same time the next relation:

$$U(\hat{\theta})=(\hat{\theta}^{(0)}-\hat{\theta})^T Q(\hat{\theta}^{(0)}-\hat{\theta})=\min.$$

The estimation which appear here are define in next way:

$$H_k(\hat{\theta}_1,\hat{\theta}_2,...,\hat{\theta}_m)=0\ ,\ k=1,2,\ldots,r$$
$$\hat{\theta}_j^{(0)}-\Delta\hat{\theta}_j^{(0)}\le\hat{\theta}_j\le\hat{\theta}_j^{(0)}+\Delta\hat{\theta}_j^{(0)}\ ,\ j=1,2,\ldots,m$$
$$U(\hat{\theta})=(\hat{\theta}^{(0)}-\hat{\theta})^T Q(\hat{\theta}^{(0)}-\hat{\theta})=\min$$

where,

$$\hat{\theta}^{(0)}=\begin{pmatrix}\hat{\theta}_1^{(0)}\\ \ldots\\ \hat{\theta}_m^{(0)}\end{pmatrix} \quad,\quad \hat{\theta}=\begin{pmatrix}\hat{\theta}_1\\ \ldots\\ \hat{\theta}_m\end{pmatrix} \quad,\quad Q=\begin{pmatrix}q_1\ldots0\\ \ldots\ldots\ldots\\ 0\ldots q_2\end{pmatrix}$$

If we note with $\hat{e}$ the errors vector define in next way $\hat{\theta}^{(0)} + \hat{e} = 0$ obtain the next result:

$$H_k\left(\hat{\theta}_1^{(0)} + \hat{e}_1, ..., \hat{\theta}_m^{(0)} + \hat{e}_m\right) = 0 \quad , \text{k=1,2,…,r}$$

$$-\Delta\hat{\theta}_j^{(0)} \le \hat{e}_j \le \Delta\hat{\theta}_j^{(0)} \quad , \text{j=1,2,…,m}$$

$$U(\hat{e}) = \hat{e}^T Q \hat{e} = \min .$$

The theorems that follows show us some properties of the estimations of the parameters.

***Theorem 1.** Let be $\theta_1, \theta_2, ..., \theta_m$ the parameters from model (1) like free parameters. If the estimators $\hat{\theta}^{(0)}{}_1, \hat{\theta}^{(0)}{}_2, ..., \hat{\theta}^{(0)}{}_m$ are resulted using minimum square method then we have:

$$E\left(\theta_j^{(0)}\right) = \theta_j \ , \ \text{j=1,2,…,m.}$$

****Theorem 2**. If the errors are normal and independents then the estimators $\hat{\theta}^{(0)}{}_1, \hat{\theta}^{(0)}{}_2, ..., \hat{\theta}^{(0)}{}_m$ which are find using minimum squares method are normally and efficiently.

After we present the problem will try to show how GAMS (General Algebric Modeling System) can be used to solve the problem. GAMS is a language which can be used to solve the problems which contain linear programming and nonlinear programming. The language is structured in four sections which are: scalars, parameters, variables and equations. The language offer the possibility to chose what kind of solve it will be used. The two possibilities are:

LP – linear programming;

NLP – nonlinear programming.

Like any language has a syntax which must be respect to write a problem correctly.

*, ** The demonstration of two theorems can be seen in [1].

We present how the problem will be write using GAMS language:

$ TITLE estimation

SCALARS

here appears the values of the coefficients of the equations (1);

PARAMETERS

θ_1 exogenous variable;

θ_2 exogenous variable;

……………………...

θ_l exogenous variable;

VARIABLES

θ_{l+1} endogenous variables;

θ_{l+2} endogenous variables;

…………………………..
θ_m endogenous variables;
EQUATIONS
eq. 0 $U(\hat{\theta}) = (\hat{\theta}^{(0)} - \hat{\theta})^T Q(\hat{\theta}^{(0)} - \hat{\theta})$;
eq. 1 $H_1(\theta_1, \theta_2, ..., \theta_m) = 0$;
eq. 2 $H_2(\theta_1, \theta_2, ..., \theta_m) = 0$;
…………………………..
eq. r $H_r(\theta_1, \theta_2, ..., \theta_m) = 0$;
eq. $Y = F(X, \theta)$;
ineq. 1 $\hat{\theta}_1^{(0)} - \Delta\hat{\theta}_1^{(0)} \le \theta_1 \le \hat{\theta}_1^{(0)} + \Delta\hat{\theta}_1^{(0)}$
ineq. 2 $\hat{\theta}_2^{(0)} - \Delta\hat{\theta}_2^{(0)} \le \theta_2 \le \hat{\theta}_2^{(0)} + \Delta\hat{\theta}_2^{(0)}$
……………………………………….
ineq. 3 $\hat{\theta}_m^{(0)} - \Delta\hat{\theta}_m^{(0)} \le \theta_m \le \hat{\theta}_m^{(0)} + \Delta\hat{\theta}_m^{(0)}$
MODEL estimation/ALL/;
SOLVE estimation MINIMIZING U USING NLP;

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