Acta Universitatis Apulensis
ISSN: 1582-5329
http://www.uab.ro/auajournal/

No. 74/2023
pp. 11-22
doi: 10.17114/j.aua.2023.74.02

KURATOWSKI CLOSURE OPERATORS IN SOFT IDEAL TOPOLOGICAL SPACES

A. Atay, F. Eren

Abstract. Obtaining a Kuratowski closure operator with the help of soft local functions is an important topic in soft ideal topological space. However, it is not possible to obtain a Kuratowski closure operator from many of these soft local functions. In order to address the lack of such an operator, the goal of this paper is to introduce another soft local function to give possibility of obtaining a Kuratowski closure operator.Also, in [12] the authors said that the closure operator can be defined by the soft semi-local function. But, in this study, with the help of an example, we showed that it cannot .

2010 *Mathematics Subject Classification*: 54A05, 54C10.

Keywords: Soft Sets, Soft Local Function, Soft Ideal Topological Spaces, Kuratowski Closure Operator.

1. Introduction

Firstly, the soft sets were introduced by Molodtsov [1]. Also, Molodtsov implemented the soft theory in many areas [1]. Then the applications of soft set have been viewed [2,3,4]. The soft topological spaces were developed [5]. The soft ideal is submitted by Kandil et al. [6]. Also, in this paper soft local function is introduced. Recently many published works made on soft local function used in soft ideal topological spaces can be found in related literature. In those paper can be mentioned among such works those aim to define such functions. In general, the researchers prefer using the generalized soft open sets instead of soft topology in soft ideal topological spaces. Regular soft local functions for the soft ideal topological spaces have been described within this work. Moreover, with the help of regular soft local functions Kuratowski closure operators $cl^{*r}_{\tilde{I}}$ and τ^{*r} topology are obtained.

2. PRELIMINARIES

Definition 1. *For* $\emptyset \neq A \subseteq E$ *and the mapping* $F : A \longrightarrow 2^X$, *a pair* $(F, A) = F_A = \left\{F(e) : e \in A \subseteq E, F : A \longrightarrow 2^X\right\}$ *is named a soft set on* X *(*E *is a parameters set,* X *is universal set and* 2^X *is power set of* X*).* $SS(X)_A$ *is a family of all these soft sets [7].*

Definition 2. *i) Null soft set (*$\tilde{\emptyset}$*) is defined as:* $F_A = \tilde{\emptyset} \Longleftrightarrow \forall e \in A, F(e) = \emptyset$.

ii) Universal soft set is defined as: $F_A = \tilde{X} \Longleftrightarrow \forall e \in A, F(e) = X$ *[8].*

Definition 3. *i)* F_A *is said to be a soft subset of* G_B *(*$F_A \tilde{\subseteq} G_B$*)*$\Longleftrightarrow A \subseteq B$ *and* $\forall e \in A, F(e) \subseteq G(e)$.

ii) F_A *and* G_B *are said to be soft equal* $\Longleftrightarrow F_A \tilde{\subseteq} G_B$ *and* $G_B \tilde{\subseteq} F_A$ *[8].*

Definition 4. *i) The difference of* F_E *and* G_E, $(F_E \backslash G_E)$ *is defined as* $(F_E \backslash G_E)(e) = F(e) \backslash G(e), \forall e \in E$, *[5].*

ii) The complement of F_A (F'_A) *is defined as:* $F' : A \longrightarrow 2^X$ *is a mapping supplied as,* $F'(e) = X \backslash F(e), \forall e \in A$. $(F'_A)' = F_A$, $\left(\tilde{X}\right)' = \tilde{\emptyset}$ *and* $\left(\tilde{\emptyset}\right)' = \tilde{X}$ *[2].*

Definition 5. *For* $x \in X, x \in (F, E) \Longleftrightarrow \forall e \in E, x \in F(e)$, *[5].*

Definition 6. *The union of* F_A *and* G_B *(*$F_A \tilde{\cup} G_B$*) is defined as [8],*

$$\forall e \in (A \cup B), (F_A \tilde{\cup} G_B)(e) = \begin{cases} F(e), & e \in A \backslash B \\ G(e), & e \in B \backslash A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

Definition 7. *The intersection of* F_A *and* G_B *(*$F_A \tilde{\cap} G_B$*) is defined as* $\forall e \in A \cap B, (F_A \tilde{\cap} G_B)(e) = F(e) \cap G(e)$ *[8].*

Definition 8. *If* $\tilde{\tau} \subseteq SS(X)_E$ *is closed under the finite intersection, arbitrary union and* $\tilde{X}, \tilde{\emptyset} \in \tilde{\tau}$, *then* $\tilde{\tau}$ *and* $(X, \tilde{\tau}, E)$ *is called a soft topology and soft topological spaces on* X, *respectively [5].*

Definition 9. *For* $(X, \tilde{\tau}, E)$,

i) F_A *is open soft set* $\Longleftrightarrow F_A \in \tilde{\tau}$, *all open soft sets is indicated by* $OS(X)$

ii) F_A *is closed soft set* $\Longleftrightarrow (F_A)' \in \tilde{\tau}$, *all closed soft sets is indicated by* $CS(X)$ *[9].*

Definition 10. *For* $(X, \tilde{\tau}, E)$, *the soft closure of* F_A *(*clF_A*) is defined as*

$clF_A = \tilde{\cap}\{G_A : F_A \tilde{\subseteq} G_A, (G_A)' \in \tilde{\tau}\}$, *[5].*

Definition 11. *For* $(X, \tilde{\tau}, E)$, *the soft interior of* F_A *(*$intF_A$*) is defined as*

$intF_A = \tilde{\cup}\{G_A : G_A \tilde{\subseteq} F_A, G_A \in \tilde{\tau}\}$, *[10].*

Definition 12. *For* $(X, \tilde{\tau}, E)$,

i) F_A *is a semi open soft* $\Longleftrightarrow F_A \tilde{\subseteq} int(clF_A)$, *all semi open soft sets is indicated by* $SOS(X)$.

ii) F_A *is a semi closed soft* $\Longleftrightarrow int(clF_A) \tilde{\subseteq} F_A$, *all semi closed soft sets is indicated by* $SCS(X)$.

ii) The semi soft closure of F_A $(sclF_A)$ *is defined as*
$sclF_A = \tilde{\cap}\{G_A : F_A \tilde{\subseteq} G_A, G_A \in SCS(X)\}$, *[9].*

Definition 13. *The soft set* F_E *is named a soft point* x_e *in* X_E *if* $\forall e \in E$ *there exist* $x \in X$ *such that* $F(e_i) = \{x\}$ *and* $F(e_j) = \emptyset$, $\forall e_j \in E \backslash \{e_i\}$. *Also, for* $e \in A$, $F(e) \subseteq G(e) \Longrightarrow x_e \tilde{\in} G_E$, *[10].*

Definition 14. *For* $(X, \tilde{\tau}, E)$, *Soft relative topology on* F_E *is defined as:*
$\tau_{F_E} = \{G_E \tilde{\cap} F_E : G_E \in \tilde{\tau}\}$. *So* $[(F, E), \tau_{F_E}, E]$ *is soft subspace of* $(X, \tilde{\tau}, E)$ *[11].*

Definition 15. *Let* $\tilde{\emptyset} \neq \tilde{I} \subseteq SS(X)_E$. *If* $\tilde{I}$ *is closed under the soft subset and finite soft union, then* $\tilde{I}$ *is named a soft ideal on* X_E *[6].*

Example 1. *For* $X = \{x_1, x_2, x_3\}$ *and* $E = \{e_1, e_2\}$ *the soft sets* $(F_1)_E, (F_2)_E, (F_3)_E, (F_4)_E, (F_5)_E$ *are soft sets described as below:*

$$F_1(e_1) = \{x_2\}, F_1(e_2) = \{x_1\},$$

$$F_2(e_1) = \{x_1, x_2\}, F_2(e_2) = \{x_1\},$$

$$F_3(e_1) = \{\}, F_3(e_2) = \{x_1\},$$

$$F_4(e_1) = \{x_1\}, F_4(e_2) = \{x_1\},$$

$$F_5(e_1) = \{x_1, x_2\}, F_5(e_2) = \{\},$$

$$F_6(e_1) = \{x_1\}, F_6(e_2) = \{\},$$

$$F_7(e_1) = \{x_2\}, F_7(e_2) = \{\},$$

Then $\tilde{I}_1 = \{\emptyset, (F_1)_E, (F_2)_E, (F_3)_E, (F_4)_E, (F_5)_E, (F_6)_E, (F_7)_E\}$*is an ideal but* $\tilde{I}_2 = \{\emptyset, (F_1)_E, (F_3)_E, (F_4)_E, (F_5)_E\}$ *isn't ideal on X.*

2.1. Soft Local Function

Definition 16. *For* $(X, \tilde{\tau}, E)$ *and a soft ideal* $\tilde{I}$*, the soft local function of* F_E *(with respect to* $\tilde{I}$ *and* $\tilde{\tau}$*) is described as:*

$F_E^*(\tilde{I}, \tilde{\tau}) = (F, E)^* = F_E^* = \tilde{\cup}\{x_e \in X_E : OS_{x_e} \tilde{\cap} F_E \notin \tilde{I}, \forall OS_{x_e} \in \tilde{\tau}\}, x_e \in OS_{x_e}$ *[6].*

Theorem 1. *For* $(X, \tilde{\tau}, E)$*, the soft sets* F_E, G_E *and two soft ideals* $\tilde{I}, \tilde{J}$*, the following is provided [6],*

i) $F_E \tilde{\subseteq} G_E \Longrightarrow F_E^* \tilde{\subseteq} G_E^*$,
ii) $\tilde{I} \subseteq \tilde{J} \Longrightarrow F_E^*(\tilde{J}) \tilde{\subseteq} G_E^*(\tilde{I})$,
iii) $F_E^* \tilde{\subseteq} clF_E$,
iv) F_E^* *is closed soft set.*
v) $(F_E^*)^* \tilde{\subseteq} F_E^*$,
vi) $[F_E \tilde{\cup} G_E]^* = F_E^* \tilde{\cup} G_E^*$,
vii) $[F_E \tilde{\cap} G_E]^* \tilde{\subseteq} F_E^* \tilde{\cap} G_E^*$,

Definition 17. *For* $(X, \tilde{\tau}, E)$*, a soft ideal* $\tilde{I}$ *and the soft closure operator* cl^* *there exists a* $*$*-soft topology* $\tilde{\tau}^* \subseteq SS(X)_E$*, finer than* $\tilde{\tau}$ *is described as*
$\tau^*(\tilde{I}) = \{F_E \in SS(X)_E : cl^*(F_E)' = (F_E)'\}$*. Also the soft topology* $\tilde{\tau}^*$ *is an unique [6].*

2.2. Soft Semi Local Function

Definition 18. *For* $(X, \tilde{\tau}, E)$ *and a soft ideal* $\tilde{I}$*, the soft semi local function of* F_E *(with respect to* $\tilde{I}$ *and* $\tilde{\tau}$*) is described as:*

$F_E^{*s}(\tilde{I}, \tilde{\tau}) = (F, E)^{*s} = F_E^{*s} = \tilde{\cup}\{x_e \in X_E : S_{x_e} \tilde{\cap} F_E \notin \tilde{I}, \forall S_{x_e} \in SOS(X)\}, x_e \in S_{x_e}$[12].

3. MAIN RESULTS

Theorem 2. *For* $(X, \tilde{\tau}, E)$*, the soft sets* F_E*,* G_E *and two soft ideals* $\tilde{I}, \tilde{J}$*, the following is provided [12].*

i) $(F, E) \tilde{\subseteq} (G, E) \Longrightarrow (F, E)^{*s} \tilde{\subseteq} (G, E)^{*s}$,
ii) $\tilde{I} \subseteq \tilde{J} \Longrightarrow (F, E)^{*s}(\tilde{J}) \tilde{\subseteq} (G, E)^{*s}(\tilde{I})$,
iii) $(F, E)^{*s} = scl(F, E)^{*s} \tilde{\subseteq} scl(F, E)$,
iv) $(F, E)^{*s}$ *is semi closed soft set*
v) $((F, E)^{*s})^{*s} \tilde{\subseteq} (F, E)^{*s}$,
vi) $[(F, E) \tilde{\cup} (G, E)]^{*s} = (F, E)^{*s} \tilde{\cup} (G, E)^{*s}$,
vii) $[(F, E) \tilde{\cap} (G, E)]^{*s} \tilde{\subseteq} (F, E)^{*s} \tilde{\cap} (G, E)^{*s}$.

Remark 1. *The above theorem 3.1./ vi. is provided in [12]. However, this is not true in general, as shown in the following example.*

Example 2. *For* $X = \{a, b, c, d\}, E = \{e_1, e_2\}$ *and the soft sets* $(F_1)_E, (F_2)_E, (F_3)_E$ *are described as in below.*

$F_1(e_1) = \{a, b\}, F_1(e_2) = \{b, d\}$
$F_2(e_1) = \{a\}, F_2(e_2) = \{b\}$
$F_3(e_1) = \{b\}, F_3(e_2) = \{d\}$
Then, $\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (F_1)_E, (F_2)_E, (F_3)_E\}$ *is soft topology and*
$SOS(X) = \{\tilde{\emptyset}, \tilde{X}, (F_1)_E, (F_2)_E, (F_3)_E, (F_2)'_E, (F_3)'_E\}$.
Let $\tilde{I} = \left\{\tilde{\emptyset}, \{(e_1, \{\}), (e_2, \{c\})\}\right\}$ *be soft ideal and* $(G_1)_E = \{(e_1, \{a\}), (e_2, \{b\})\}, (G_2)_E = \{(e_1, \{b\}), (e_2, \{d\})\}$ *be soft sets on* X*. Then,*
$(G_1)^{*s}_E = \{(e_1, \{a\}), (e_2, \{b\})\}$,
$(G_2)^{*s}_E = \{(e_1, \{b\}), (e_2, \{d\})\}$,
$(G_1 \cup G_2)^{*s}_E = \tilde{X}$.
So, $(G_1)^{*s}_E \tilde{\cup} (G_2)^{*s}_E \neq (G_1 \cup G_2)^{*s}_E$.

Remark 2. *Because of above Remark we are not able to define a Closure operator with the help of soft semi local function. So, cannot generated a new soft topology with soft semi local functions.*

3.1. Soft Regular Local Function

Definition 19. *For* $(X, \tilde{\tau}, E)$,

i) F_A *is a regular open soft* $\Longleftrightarrow F_A = int\,(clF_A)$, *all regular open soft sets is indicated by* $ROS(X)$.

ii) F_A *is a regular closed soft* $\Longleftrightarrow F_A = cl\,(intF_A)$, *all regular closed soft sets is indicated by* $RCS(X)$.

iii) The regular soft closure of $F_A\,(rclF_A)$ *is defined as:*
$rclF_A = \tilde{\cap}\left\{G_A : F_A \tilde{\subseteq} G_A, G_A \in RCS\,(X)\right\}$,

iv) For a soft ideal $\tilde{I}$, $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *is named a soft regular ideal space.*

Definition 20. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$, *the soft regular local function of* F_E *(with respect to* $\tilde{I}$ *and* $\tilde{\tau}$*) is described as:*

$$F^{*r}_E\left(\tilde{I}, \tilde{\tau}\right) = F^{*r}_E = \tilde{\cup}\left\{x_e \in X_E : R_{x_e} \tilde{\cap} F_E \notin \tilde{I}, \forall R_{x_e} \in ROS(X)\right\},\ x_e \in R_{x_e}.$$

Theorem 3. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft set* F_E, *the following is provided.*

i) $F^{*s}_E \tilde{\subseteq} F^{*}_E \tilde{\subseteq} F^{*r}_E$,
ii) $F^{*}_E = F^{*r}_E$ *if* $OS(X) = ROS(X)$,
iii) If $F_E \in \tilde{I}$, *then* $F^{*r}_E = \tilde{\emptyset}$,
iv) $\tilde{\emptyset}^{*r} = \tilde{\emptyset}$.

Proof. i) Let $x_e \in F_E^{*s}$. Then, $S_{x_e} \tilde{\cap} F_E \notin \tilde{I}$ for every $S_{x_e} \in SOS(X)$. Since every open soft set is semi open soft set, therefore $x_e \in F_E^{*}$. Because of every regular open soft set is open soft set, $x_e \in F_E^{*r}$. Converse is not true in general; it is shown in Example 4.2

ii) It is obvious from definition of soft regular local and soft semi local functions.

iii) Let $F_E \in \tilde{I}$ and $x_e \in F_E^{*r}$. Then for every regular open soft set R_{x_e} containing x_e, $R_{x_e} \tilde{\cap} F_E \notin \tilde{I}$. On the other hand, $\tilde{X}$ is also regular open soft set. So $\tilde{X} \tilde{\cap} F_E = F_E \notin \tilde{I}$. It is contradiction.

iv) Because of iii. it is obvious.

Example 3. *Let $X = \{a, b, c, d\}$ and $E = \{e_1, e_2, e_3\}$. $(F_1)_E, (F_2)_E, (F_3)_E$ and G_E be soft sets on X, which describe in below,*

$$F_1(e_1) = \{a, b, c\}, F_1(e_2) = \{a, b\}, F_1(e_3) = \{a\},$$

$$F_2(e_1) = \{a\}, F_2(e_2) = \{\}, F_2(e_3) = \{\},$$

$$F_3(e_1) = \{a\}, F_3(e_2) = \{a, b\}, F_3(e_3) = \{\},$$

$$G(e_1) = \{a\}, G(e_2) = \{b\}, G(e_3) = \{c\}$$

Then, $\tau = \{\tilde{\emptyset}, \tilde{X}, (F_1)_E, (F_2)_E, (F_3)_E\}$is a soft topology on X, $SOS(X) = \{\tilde{\emptyset}, \tilde{X}, (F_1)_E, (F_2)_E, (F_3)_E, (F_2)'_E, (F_3)'_E\}$ and $ROS(X) = \{\tilde{\emptyset}, \tilde{X}, (F_2)_E, (F_3)_E\}$. Also, $\tilde{I} = \{\tilde{\emptyset}, I_1, I_2, I_3\}$ is a soft ideal with

$$(I_1, E) = \{(e_1, \{b\}), (e_2, \{c\}), (e_3, \{\})\},$$

$$(I_2, E) = \{(e_1, \{b\}), (e_2, \{\}), (e_3, \{\})\},$$

$$(I_3, E) = \{(e_1, \{\}), (e_2, \{c\}), (e_3, \{\})\}.$$

Then,

$$G_E^{*s} = \{(e_1, \{a\}), (e_2, \{a, b\}), (e_3, \{b, c, d\})\},$$

,

$$G_E^{*} = \{(e_1, \{a, b, c\}), (e_2, \tilde{X}), (e_3, \{b, c, d\})\},$$

$$G_E^{*r} = \{(e_1, \{a,b,d\}), (e_2, \tilde{X}), (e_3, \tilde{X})\}.$$

So, $G_E^{*r} \tilde{\nsubseteq} G_E^{*}$.

Theorem 4. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft set* F_E, G_E, *the following is provided.*

i) If $F_E \tilde{\subseteq} G_E$, *then* $F_E^{*r} \tilde{\subseteq} G_E^{*r}$,

ii) If $\tilde{I}, \tilde{J}$ *soft ideal on* X_E *and* $\tilde{I} \tilde{\subseteq} \tilde{J}$, *then* $F_E^{*r}\left(\tilde{J}\right) \tilde{\subseteq} F_E^{*r}\left(\tilde{I}\right)$.

Proof. i) Let $x_e \in F_E^{*r}$. Then, $R_{x_e} \tilde{\cap} F_E \notin \tilde{I}, \forall R_{x_e} \in ROS(X)$. Since $R_{x_e} \tilde{\cap} F_E \tilde{\subseteq} R_{x_e} \tilde{\cap} G_E$, then $R_{x_e} \tilde{\cap} G_E \notin \tilde{I}$.

ii) Let $x_e \in F_E^{*r}\left(\tilde{J}\right)$. Then, $R_{x_e} \tilde{\cap} F_E \notin \tilde{J}, \forall R_{x_e} \in ROS(X)$. Since $\tilde{I} \tilde{\subseteq} \tilde{J}$, then $R_{x_e} \tilde{\cap} F_E \notin \tilde{I}$.

Theorem 5. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft set* F_E, G_E, *the following is provided.*

i) $F_E^{*r} = clF_E^{*r} \tilde{\subseteq} rclF_E^{*r}$ *and* F_E^{*r} *is soft closed in* $(X, \tilde{\tau}, E)$,

ii) $(F_E^{*r})^{*r} \tilde{\subseteq} F_E^{*r}$,

iii) $F_E^{*r} \tilde{\cup} G_E^{*r} = (F_E \tilde{\cup} G_E)^{*r}$,

iv) $(F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} F_E^{*r} \tilde{\cap} G_E^{*r}$,

v) $F_E^{*r} \backslash G_E^{*r} = (F_E \backslash G_E)^{*r} \backslash G_E^{*r} \tilde{\subseteq} (F_E \backslash G_E)^{*r}$.

Proof. i) In general, we know $F_E^{*r} \tilde{\subseteq} clF_E^{*r}$. Let $x_e \in clF_E^{*r}$.Then $F_E^{*r} \tilde{\cap} T_E \neq \tilde{\emptyset}$, $T_E \in \tilde{\tau}(X_E, x_e)$. Also given the soft set $R_E \in ROS(X_E, x_e)$. Since R_E is soft open, $F_E^{*r} \tilde{\cap} R_E \neq \tilde{\emptyset}$. Therefore, there exist some $y_e \in F_E^{*r} \tilde{\cap} R_E$ and $R_E \in ROS(X_E, y_e)$. Since $y_e \in F_E^{*r}$, $R_E \tilde{\cap} F_E \notin \tilde{I}$ and hence $x_e \in F_E^{*r}$. Hence we have $clF_E^{*r} \tilde{\subseteq} F_E^{*r}$. So $F_E^{*r} = clF_E^{*r}$. Again, let $x_e \in F_E^{*r} = clF_E^{*r}$, then $R_E \tilde{\cap} F_E \notin \tilde{I}$, for every $R_E \in ROS(X_E, x_e)$. This implies $F_E^{*r} \tilde{\cap} R_E \neq \tilde{\emptyset}$ for every $R_E \in ROS(X_E, x_e)$. Therefore, $x_e \in rclF_E^{*r}$. This shows that $F_E^{*r} = clF_E^{*r} \tilde{\subseteq} rclF_E^{*r}$. Since $F_E^{*r} = clF_E^{*r}$, F_E^{*r} is soft closed.

ii) Let $x_e \in (F_E^{*r})^{*r}$. Then for every $R_E \in ROS(X_E, x_e)$, $R_E \tilde{\cap} F_E^{*r} \notin \tilde{I}$ and hence $R_E \tilde{\cap} F_E^{*r} \neq \tilde{\emptyset}$. Let $y_e \in R_E \tilde{\cap} F_E^{*r}$. Then $R_E \in ROS(X_E, y_e)$ and $y_e \in F_E^{*r}$. Hence we have $R_E \tilde{\cap} F_E \notin \tilde{I}$ and $x_e \in F_E^{*r}$. This shows that $(F_E^{*r})^{*r} \tilde{\subseteq} F_E^{*r}$.

iii) By theorem 3.3. (i), we have $F_E^{*r} \tilde{\cup} G_E^{*r} \tilde{\subseteq} (F_E \tilde{\cup} G_E)^{*r}$. To prove the reverse inclusion, let $x_e \notin F_E^{*r} \tilde{\cup} G_E^{*r}$. Then x_e belongs neither to F_E^{*r} nor to G_E^{*r}. Therefore there exist $U_E, V_E \in ROS(X_E, x_e)$ such that $F_E \tilde{\cap} U_E \in \tilde{I}$ and $G_E \tilde{\cap} V_E \in \tilde{I}$. Since $\tilde{I}$ is closed under operation of union, $(F_E \tilde{\cap} U_E) \tilde{\cup} (G_E \tilde{\cap} V_E) \in \tilde{I}$.

$(F_E \tilde{\cap} U_E) \tilde{\cup} (G_E \tilde{\cap} V_E) = [(F_E \tilde{\cap} U_E) \tilde{\cup} V_E] \tilde{\cap} [(F_E \tilde{\cap} U_E) \tilde{\cup} G_E]$
$= (U_E \tilde{\cup} V_E) \tilde{\cap} (F_E \tilde{\cup} V_E) \tilde{\cap} (G_E \tilde{\cup} U_E) \tilde{\cap} (F_E \tilde{\cup} G_E)$

On the other hand since
$V_E \tilde{\subseteq} (F_E \tilde{\cup} V_E), U_E \tilde{\subseteq} (G_E \tilde{\cup} U_E)$ and $(U_E \tilde{\cap} V_E) \tilde{\subseteq} (U_E \tilde{\cup} V_E)$ we have
$(U_E \tilde{\cup} V_E) \tilde{\cap} (F_E \tilde{\cup} V_E) \tilde{\cap} (G_E \tilde{\cup} U_E) \tilde{\cap} (F_E \tilde{\cup} G_E) \supseteq (U_E \tilde{\cap} V_E) \tilde{\cap} (F_E \tilde{\cup} G_E)$

Since $\tilde{I}$ is closed under operation of subset, $(U_E \tilde{\cap} V_E) \tilde{\cap} (F_E \tilde{\cup} G_E) \in \tilde{I}$. Also we know that regular open soft sets closed under the finite intersections,

$U_E \tilde{\cap} V_E \in ROS(X_E, x_e)$ and so $x_e \notin (F_E \tilde{\cup} G_E)^{*r}$.

Hence $\left(\tilde{X} \setminus F_E^{*r}\right) \tilde{\cap} \left(\tilde{X} \setminus G_E^{*r}\right) \tilde{\subseteq} \tilde{X} \setminus (F_E \tilde{\cup} G_E)^{*r}$ or $(F_E \tilde{\cup} G_E)^{*r} \tilde{\subseteq} F_E^{*r} \tilde{\cup} G_E^{*r}$.

iv) By theorem 3.3. (i) $(F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} F_E^{*r}$ and $(F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} G_E^{*r}$ so $(F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} F_E^{*r} \tilde{\cap} G_E^{*r}$.

v) We have by theorem 3.4. (iii),
$F_E^{*r} = [(F_E \setminus G_E) \tilde{\cup} (F_E \tilde{\cap} G_E)]^{*r} = (F_E \setminus G_E)^{*r} \tilde{\cup} (F_E \tilde{\cap} G_E)^{*r}$. Thus
$F_E^{*r} \setminus G_E^{*r} = (F_E \setminus G_E)^{*r} \setminus G_E^{*r}$.
On the other hand, by theorem 3.3. (i),$(F_E \setminus G_E)^{*r} \tilde{\subseteq} F_E^{*r}$ and hence
$(F_E \setminus G_E)^{*r} \setminus G_E^{*r} \tilde{\subseteq} F_E^{*r} \setminus G_E^{*r}$ Hence $F_E^{*r} \setminus G_E^{*r} = (F_E \setminus G_E)^{*r} \setminus G_E^{*r} \tilde{\subseteq} (F_E \setminus G_E)^{*r}$.

Theorem 6. *For $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ and the soft set F_E, G_E, the following is provided.*
i) If $\tilde{I}_0 \in \tilde{I}$, then $\left(F_E \setminus \tilde{I}_0\right)^{*r} = F_E^{*r} = \left(F_E \tilde{\cup} \tilde{I}_0\right)^{*r}$,
ii) If $G_E \tilde{\subseteq} X_E$, then $G_E \tilde{\cap} (F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} G_E \tilde{\cap} F_E^{*r}$,
iii) If $G_E \in ROS(X_E, x_e)$, then $G_E \cap F_E \in \tilde{I} \Longrightarrow G_E \tilde{\cap} F_E^{*r} = \emptyset$,
iv) $(F_E^{*r} \tilde{\cap} F_E)^{*r} \tilde{\subseteq} F_E^{*r}$.

Proof. i) Since $\tilde{I}_0 \in \tilde{I}$, by theorem 3.2. (iii) $\tilde{I}_0^{*r} = \emptyset$. By theorem 3.4. (v), $F_E^{*r} = \left(F_E \setminus \tilde{I}_0\right)^{*r}$ and by theorem 3.4. (iii), $\left(F_E \tilde{\cup} \tilde{I}_0\right)^{*r} = F_E^{*r} \tilde{\cup} \tilde{I}_0^{*r} = \emptyset \tilde{\cup} F_E^{*r} = F_E^{*r}$.

ii) Since $G_E \tilde{\cap} F_E \tilde{\subseteq} F_E$, by theorem 3.3. (i), $(F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} F_E^{*r}$ and hence $G_E \tilde{\cap} (F_E \tilde{\cap} G_E)^{*r} \tilde{\subseteq} G_E \tilde{\cap} F_E^{*r}$.

iii) Let $F_E \tilde{\cap} G_E \in \tilde{I}$, then for every $x_e \in G_E, x_e \notin F_E^{*r}$ because of $G_E \in ROS(X_E, x_e)$. So $G_E \tilde{\cap} F_E^{*r} = \emptyset$.

iv) By theorem 3.3. (i) $(F_E^{*r} \tilde{\cap} F_E)^{*r} \tilde{\subseteq} (F_E^{*r})^{*r}$. On the other hand, from theorem 3.4. (ii) we have $(F_E^{*r} \tilde{\cap} F_E)^{*r} \tilde{\subseteq} (F_E^{*r})^{*r} \tilde{\subseteq} F_E^{*r}$.

3.2. A New Operator via Soft Regular Local Function

Remark 3. *We are able to define a closure operator with the help of soft regular local function. Because the $()^{*r}$ operator satisfy the conditions of Theorem 3.2. (iv),*

Theorem 3.4. (ii) and Theorem 3.1.3. (iii). Thus $cl_{\tilde{I}}^{*r} : SS(X)_E \to SS(X)_E$ *defined by*
$cl_{\tilde{I}}^{*r} F_E = F_E \tilde{\cup} F_E^{*r}, \forall F_E \in SS(X)_E$ *is a Kuratowski closure operator. Hence it generates a* $\tilde{\tau}^{*r}$ *topology:* $\tilde{\tau}^{*r}\left(\tilde{I}\right) = \{F_E \in SS(X)_E : cl_{\tilde{I}}^{*r}(F_E)' = (F_E)'\}$.

Definition 21. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft set* F_E *an operator* $\psi_{*r} : \mathcal{P}(X) \to \tau$ *is defined as:* $\psi_{*r}(F_E) = \{x \in X : \exists U_E \in ROS(X, x), U_E \setminus F_E \in \tilde{I}\}$*, for every* $F_E \in \mathcal{P}(X)$*. Also we know that* $\psi_{*r}(F_E) = X \setminus (X \setminus F_E)^{*r}$.

Theorem 7. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft sets* F_E, G_E,
i) $int(F_E^{*r}) \tilde{\subseteq} \psi_{*r}(F_E)$,
ii) $\psi_{*r}(F_E)$ *is open,*
iii) If $F_E \tilde{\subseteq} G_E$ *than* $\psi_{*r}(F_E) \tilde{\subseteq} \psi_{*r}(G_E)$,
iv) $\psi_{*r}(F_E) \tilde{\cup} \psi_{*r}(G_E) \tilde{\subseteq} \psi_{*r}(F_E \tilde{\cup} G_E)$,
v) $\psi_{*r}(F_E \tilde{\cap} G_E) = \psi_{*r}(F_E) \tilde{\cap} \psi_{*r}(G_E)$,
vi) $\psi_{*r}(F_E) \tilde{\subseteq} \psi(F_E)$.

Proof. i) $\psi_{*r}(F_E) = X \setminus (X \setminus F_E)^{*r} \tilde{\supseteq} X \setminus cl(X \setminus F_E)^{*r}$. So $\psi_{*r}(F_E) \tilde{\supseteq} int(F_E^{*r})$.

ii) We know that, for any soft set, the soft regular local function is a closed, so $(X \setminus F_E)^{*r})$ is closed. So $X \setminus (X \setminus F_E)^{*r}) = \psi_{*r}(F_E)$ is a open set.
iii) $F_E \tilde{\subseteq} G_E \Rightarrow X \setminus G_E \tilde{\subseteq} X \setminus F_E \Rightarrow (X \setminus G_E)^{*r} \tilde{\subseteq} (X \setminus F_E)^{*r} \Rightarrow X \setminus (X \setminus F_E)^{*r} \tilde{\supseteq} X \setminus (X \setminus G_E)^{*r} \Rightarrow \psi_{*r}(F_E) \tilde{\subseteq} \psi_{*r}(G_E)$

iv) Proof is obvious from (iii).
v) $\psi_{*r}(F_E \tilde{\cap} G_E) = X \setminus (X \setminus (F_E \tilde{\cap} G_E))^{*r} = X \setminus [(X \setminus F_E) \tilde{\cup} (X \setminus G_E)]^{*r} = X \setminus [(X \setminus F_E)^{*r} \tilde{\cup} (X \setminus G_E)^{*r}] = [X \setminus (X \setminus F_E)^{*r}] \tilde{\cap} [X \setminus (X \setminus G_E)^{*r}] = \psi_{*r}(F_E) \tilde{\cap} \psi_{*r}(G_E)$.

(vi) We have that $(X \setminus F_E)^{*} \tilde{\subseteq} (X \setminus F_E)^{*r} \Rightarrow X \setminus (X \setminus F_E)^{*r} \tilde{\subseteq} X \tilde{\subseteq} (X \setminus F_E)^{*} \Rightarrow \psi_{*r}(F_E) \tilde{\subseteq} \psi(F_E)$.

Theorem 8. *For* $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ *and the soft sets* F_E, G_E,
i) $\psi_{*r}(F_E) = \psi_{*r}(\psi_{*r}(F_E)) \Leftrightarrow (X \setminus F_E)^{*r} = [(X \setminus F_E)^{*r}]^{*r}$,
ii) If $I_0 \in \tilde{I}$*, then* $\psi_{*r}(F_E \setminus I_0) = \psi_{*r}(F_E)$,
iii) If $I_0 \in \tilde{I}$*, then* $\psi_{*r}(F_E \cup I_0) = \psi_{*r}(F_E)$,
iv) If $(F_E \cup G_E) \setminus (F_E \cap G_E) \in \tilde{I}$ *then* $\psi_{*r}(F_E) = \psi_{*r}(G_E)$,
v) If $F_E \in ROS(X, \tilde{\tau})$ *then* $F_E \subseteq \psi_{*r}(F_E)$.

Proof. i) From definition of $psi_{*r}(F_E)$ proof is obvious.

ii) $\psi_{*r}(F_E \setminus I_0) = X \setminus [X \setminus (F_E \setminus I_0)]^{*r} = X \setminus [(X \setminus F_E)\tilde{\cup} I_0]^{*r} = X \setminus [X \setminus F_E]^{*r} = \psi_{*r}(F_E)$,
iii) $\psi_{*r}(F_E \cup I_0) = X \setminus [X \setminus (F_E \tilde{\cup} I_0)]^{*r} = X \setminus [(X \setminus F_E) \setminus I_0]^{*r} = X \setminus [X \setminus F_E]^{*r} = \psi_{*r}(F_E)$,
iv) $(F_E \tilde{\cup} G_E) \setminus (F_E \tilde{\cap} G_E) \in \tilde{I} \Rightarrow (F_E \setminus G_E)\tilde{\cup}(G_E \setminus F_E) \in \tilde{I}$. Let $F_E \setminus G_E = I_1$ and $G_E \setminus F_E = I_2$. Hence $G_E = (F_E \setminus I_1)\tilde{\cup} I_2$. Thus $\psi_{*r}(F_E) = \psi_{*r}(F_E \setminus I_1) = \psi_{*r}((F_E \setminus I_1)\tilde{\cup} I_2) = \psi_{*r}(G_E)$,
v) Since $F_E \in ROS(X, \tilde{\tau})$, $X \setminus F_E \in RCS(X, \tilde{\tau})$. So $(X \setminus F_E) = rcl(X\ F_E)$. Hence $(X \setminus F_E)^{*r} \tilde{\subseteq} rcl(X \setminus F_E) = X \setminus F_E \Rightarrow (X \setminus F_E)^{*r} \tilde{\subseteq} X \setminus F_E \Rightarrow F_E \tilde{\subseteq} X \setminus (X \setminus F_E)^{*r} \Rightarrow F_E \subseteq \psi_{*r}(F_E)$.

Theorem 9. *For $(X, ROS(X, \tilde{\tau}, E), \tilde{I})$ and the soft sets F_E,*
(i) $psi_{*r}(F_E) = \tilde{\bigcup}\{U \in ROS(X, \tilde{\tau}, E) : U \setminus F_E \in \tilde{I}\}$.
(ii) $psi_{*r}(F_E) \supseteq \tilde{\bigcup}\{U \in ROS(X, \tilde{\tau}, E) : (U \setminus F_E) \cup (F_E \setminus U) \in \tilde{I}\}$.

Proof. i) Proof is obvious from definition of $\psi_{*r}(F_E)$.
ii) Since I is heredity, we have $\tilde{\bigcup}\{U \in ROS(X, \tilde{\tau}, E) : (U \setminus F_E)\tilde{\cup}(F_E \setminus U) \in \tilde{I}\} \tilde{\subseteq} \tilde{\bigcup}\{U \in ROS(X, \tilde{\tau}, E) : (U \setminus F_E) \in \tilde{I}\} = \psi_{*r}(F_E)$.

4. DISCUSSION

As in topological spaces, soft closure operators, can be defined with the help of soft local functions, is an important detail in soft ideal topological space [13]. In this study we will show that this is not always possible. For example closure operator can be defined by the soft local function [6] and by the soft θ-local function [14]. Also, in [12] they said that the closure operator can be defined by the soft semi-local function. But, in this study, with the help of an example, we showed that it cannot . Therefore, a new soft topology cannot be defined from the original one with the help of soft semi-local function. Also, we will define a new soft local function called soft regular local function. And the closure operator can be defined by the soft regular local function. So, a new soft topology can be defined from the original one with the help of soft regular local function.

References

[1] Molodtsov D. A. 1999. Soft set theory-first results, Computers and Mathematics with Applications, 37(4-5), 19-31. Doi: 10.1016/S0898-1221(99)00056-5

[2] Ali M. I., Feng F., Liu X., Min W. K. and Shabir M. 2009. On some new operations in soft set theory, Computers and Mathematics with Applications, 57(9), 1547-1553. Doi: 10.1016/j.camwa.2008.11.009

[3] Kovkov D. V., Kolbanov V. M. and Molodtsov D. A. 2007. Soft sets theory-based optimization, Journal of Computer and Systems Sciences International, 46(6), 872-880. Doi: 10.1134/S1064230707060032

[4] Pei D. and Miao D. 2005. From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager,B. Zhang (Eds.), Proceedings of Granular Computing, in:IEEE, 2, 617-621.

[5] Shabir M. and Naz M. 2011. On soft topological spaces, Comput. Math. Appl., 61(7), 1786-1799. 10.1016/j.camwa.2011.02.006

[6] Kandil, A., Tantawy, O. A. E., El-Sheikh S. A. and Abd El-latif, A. M. 2014. Soft Ideal Theory Soft Local Function and Generated Soft Topological Spaces, Applied Mathematics & Information Sciences, 8(4): 1595-1603. Doi: 10.12785/amis/080413

[7] Maji, P. K., Biswas R. and Roy A. R. 2003. Soft set theory, Computers and Mathematics with Applications, 45(4-5), 555-562(2003). Doi: 10.1016/S0898-1221(03)00016-6

[8] Zorlutuna, I., Akdag, M., Min W.K. and Atmaca S. 2012. Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 3(2), 171-185. . http://www.afmi.or.kr

[9] Sujoy Das and Samanta S. K. 2013. Soft metric, Annals of Fuzzy Mathematics and Informatics, 6(1), 77-94. http://www.afmi.or.kr

[10] Hussain S.and Ahmad B. 2011. Some properties of soft topological spaces, Comput. Math. Appl., 62(11), 4058-4067. Doi: 10.1016/j.camwa.2011.09.051

[11] Jankovic, D. and Hamlet, T.R. 1990. New topologies from old via ideals, The American Mathematical Monthly, 97: 295-310. Doi: 10.1080/00029890.1990.11995593

[12] Gharib, F. A., Abd El-latif, A. M. 2019. Soft semi local functions in soft ideal topological spaces, European Journal of Pure and Applied Mathematics, 12(3): 857-869. Doi: 10.29020/nybg.ejpam.v12i3.3442

[13] Atay, A., Tutalar, H. I. 2018. A note on the new set operator ψ_r, Tbilisi Mathematical Journal, 11(4): 43–52. Doi: 10.32513/tbilisi/1546570884

[14] Al-Omari A. 2019. Soft topology in ideal topological spaces, Hacettepe Journal of Mathematics & Statistics, 48(5), 1277-1285. Doi: 10.15672/HJMS.2018.557

Arife ATAY
Department of Mathematics, Faculty of Science,

University of Dicle,
Diyarbakır, Türkiye
email: *arifea@dicle.edu.tr*

Fırat EREN
Institute of science,
University of Dicle,
Diyarbakır, Türkiye
email: *erenfirat21@gmail.com*