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SPATIAL DISCRETIZATION OF AN IMPULSIVE COHEN-GROSSBERG NEURAL NETWORK WITH TIME-VARYING AND DISTRIBUTED DELAYS AND REACTION-DIFFUSION TERMS

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Abstract

In this paper we consider a straight homogeneous elastic rod with finite length and constant section, supported at the ends and lying on an elastic foundation of Winkler type. On the rod act uniformly distributed loads, as well as two concentrated loads and moments. The reactions at the supported ends, as well as the deflection of the rod are given. The influences of the concentrated loads and of the moment on the reactions are studied and shown in charts.

1 Introduction

In the study of the bending of elastic rod on elastic foundation we come across difficulties owing to the following factors:

1. the rod is loaded with discontinuous loads;

2. the action of some concentrated loads and moments;

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3. discontinuities of the mechanical properties of the rod and of the elastic foundation.

The general and unitary method to deal with the problems concerning external discontinuities (e.g. discontinuous loading) and internal discontinuities (e.g. discontinuous loading) is the distribution theory.

In the framework of the theory a single equation which contains the boundary and jump conditions is obtained.

The distribution theory was used in [2], [3], [7], [8], [9] and [10] for analyzing beams with discontinuities.

A bending problem with discontinuities in which the distribution theory isn't systematically applied, being a combination between classical mathematical analysis and the distribution theory, is studied in [11].

In [2] and [7] using the distribution theory in a systematic manner the bending problem with discontinuities of a finite elastic rod on elastic foundation under the action of concentrated loads are studied.

We mention that the advantaged of this method is that it gives the general expression of the deflection irrespective of the type of loading of the rod, which allows a global analysis of the influence of each individual load.

In this paper we consider a straight homogeneous elastic rod with finite length and constant section, supported at the ends and lying on an elastic foundation of Winkler type (the reaction of the elastic foundation is proportional to the deflection of the rod in that point and is independent from the deflection of other parts of the foundation).

On the rod, some uniformly distributed loads act as well as two concentrated loads and moments.

The reactions at the supported ends, as well as the deflection of the rod are given. The influence of the flexural stiffness of the rod and the rigidity coefficient of the elastic foundation on the deflection of the rod is analyzed. The influences of the concentrated loads and of the moment on the reactions are studied and shown in charts.

2 The study of the bending of the finite elastic rod

Let be OA a straight homogeneous elastic rod of finite length ℓ and with constant cross-section, supported in the points O and A, which lies on an elastic foundation of Winkler type [1].

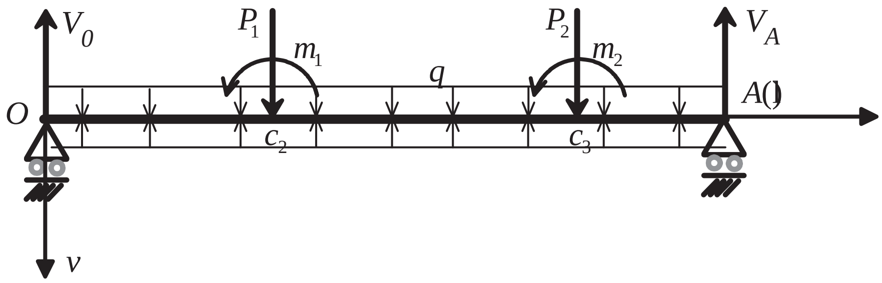

Fig. 1. Elastic rod supported on an elastic foundation

We admit that on the rod act uniformly distributed loads of intensity q, as well as concentrated loads of value P_1 and P_2 applied in point $c_1 \in (0, \ell)$, $c_2 \in (0, \ell)$, $c_1 < c_2$, respectively.

We shall denote by $v(x)$, $x \in [0, \ell]$ the deflection of the rod. We denote by $\tilde{\partial}_x = \frac{\tilde{d}}{dx}$, $\partial_x = \frac{d}{dx}$ the derivative in classic sense and the derivative in distribution sense, respectively.

For a Winkler model, it is assumed that the reaction of the elastic foundation $q_e(x)$, $x \in [0, \ell]$ exerted on the rod is proportional to its deflection at that point and is independent from the deflection of other parts of the foundation hence

$$q_e(x) = -kv(x), \qquad x \in [0, \ell] \tag{1}$$

where k is called the rigidity coefficient of the elastic foundation.

We shall denote by $D'(\mathbb{R})$ the distribution (continuous linear functional) defined on the test functions space $D(\mathbb{R})$, which are indefinite derivable functions with compact support.

We denote by $D'_+ \subset D'(\mathbb{R})$ the distributions from $D'(\mathbb{R})$ having the supports on $[0, \infty)$. We mention that the distributions from D'_+ represent a convolution algebra without divisors of zero.

We observe that $\tilde{v}(x) = \begin{cases} v(x), & x \in [0, \ell] \\ 0, & x \notin [0, \ell] \end{cases}$ represents a function type distribution from D'_+, because its support is in $[0, \ell] \subset [0, \infty)$.

We denote by the symbol $[\ ]_a$ the jump of a certain value at point $x = a$.

Due to the way in which the rod is fixed the boundary conditions are

$$\begin{gathered} \tilde{v}(0+0) = 0,\ \tilde{v}(0-0) = 0,\ \tilde{v}(\ell+0) = 0,\ \tilde{v}(\ell-0) = 0, \\ \tilde{\partial}_x^2 \tilde{v}(0+0) = 0,\ \tilde{\partial}_x^2 \tilde{v}(0-0) = 0,\ \tilde{\partial}_x^2 \tilde{v}(\ell+0) = 0,\ \tilde{\partial}_x^2 \tilde{v}(\ell-0) = 0. \end{gathered} \tag{2}$$

From the boundary conditions (2) we have

$$\begin{gathered}[\tilde{v}]_0=\tilde{v}(0+0)-\tilde{v}(0-0)=0,\ [\tilde{v}]_\ell=\tilde{v}(\ell+0)-\tilde{v}(\ell-0)=0,\\ \left[\tilde{\partial}_x\tilde{v}\right]_0=\tilde{v}'(0+0)-\tilde{v}'(0-0)=\tilde{v}'(0+0),\\ \left[\tilde{\partial}_x\tilde{v}\right]_\ell=\tilde{v}'(\ell+0)-\tilde{v}'(\ell-0)=-\tilde{v}'(\ell-0).\end{gathered}\tag{3}$$

According to [2] for the deflection $\tilde{v}$ we have the expression

$$\tilde{v}(x)=\begin{cases}0, & x\notin[0,\,\ell]\\ \frac{q}{4EI\omega^3}\int\limits_a^x u(x-t)dt+ & \\ +\frac{1}{4EI\omega^3}\sum\limits_{i=1}^4 P_iH(x-c_i)u(x-c_i)+ & \\ +\frac{1}{4EI\omega^3}\sum\limits_{i=1}^4 m_iH(x-c_i)u_1(x-c_i)+ & x\in[0,\,\ell]\\ +\frac{1}{4\omega^3}[\tilde{v}]_0H(x-a)u_3(x-a)+ & \\ +\frac{1}{4\omega^3}\left[\tilde{\partial}_x\tilde{v}\right]_\ell H(x-a)u_2(x-a), & \end{cases}\tag{4}$$

where $\omega=\sqrt[4]{\frac{k}{4EI}}$, H is the Heaviside function, $P_1=-V_0$, $P_4=-V_A$, $m_1=m_4=0$.

We mention that we introduce the real-valued functions $u,\ u_1,\ u_2,\ u_3\in C^\infty(\mathbb{R})$ having the expressions:

$$\begin{aligned}&u(x)=\cosh\omega x\sin\omega x-\sinh\omega x\cos\omega x,\\ &u_1(x)=u'(x)=2\omega\sinh\omega x\sin\omega x,\\ &u_2(x)=u''(x)=2\omega^2\left(\cosh\omega x\sin\omega x+\sinh\omega x\cos\omega x\right),\\ &u_3(x)=u'''(x)=4\omega^3\left(\cosh\omega x\cos\omega x\right).\end{aligned}\tag{5}$$

We have

$$u^4(x)=u_3'(x)=-4\omega^4u(x)$$

From here results

$$u^{(4k)}(x)=\left(-4\omega^4\right)^ku(x),\qquad u^{(4k+1)}(x)=\left(-4\omega^4\right)^ku_1(x),$$

$$u^{(4k+2)}(x)=\left(-4\omega^4\right)^ku_2(x),\qquad u^{(4k+3)}(x)=\left(-4\omega^4\right)^ku_3(x).$$

Because any natural number $n\geq 4$ can be written under the form $n=4k+p$, $p=0,\,1,\,2,\,3,\ k\in\mathbb{N}$, we have:

Any $n\geq 4$ order derivative of the function $u\in C^\infty(\mathbb{R})$ represents a multiple of one of the functions u, $u_1=u'$, $u_2=u''$, $u_3=u'''$, namely

$$u^{(n)}(x) = \begin{cases} \left(-4\omega^4\right)^k u(x),\ n = 4k \\ \left(-4\omega^4\right)^k u_1(x),\ n = 4k+1 \\ \left(-4\omega^4\right)^k u_2(x),\ n = 4k+2 \\ \left(-4\omega^4\right)^k u_3(x),\ n = 4k+3 \end{cases} \quad k = 1,\, 2,\, 3, \ldots$$

Using the formula $\int\limits_0^x f(x-t)dt = \int\limits_0^x f(t)dt$ the deflection $\tilde{v}$ can be written under the form

$$\tilde{v}(x) = \begin{cases} \frac{q}{4EI\omega^3}\int\limits_0^x u(t)dt - \frac{V_0 u(x)}{4EI\omega^3} + \frac{\tilde{v}'(0+0)u_2(x)}{4\omega^3}, \quad x \in [0,\, c_2) \\ \begin{array}{c} \frac{q}{4EI\omega^3}\int\limits_0^x u(t)dt - \frac{V_0 u(x)}{4EI\omega^3} + \frac{\tilde{v}'(0+0)u_2(x)}{4\omega^3} + \\ + \frac{1}{4EI\omega^3}\left[P_2 u\left(x-c_2\right) + m_2 u_1\left(x-c_2\right)\right] \end{array}, \quad x \in [c_2,\, c_3) \\ \begin{array}{c} \frac{q}{4EI\omega^3}\int\limits_0^x u(t)dt - \frac{V_0 u(x)}{4EI\omega^3} + \frac{\tilde{v}'(0+0)u_2(x)}{4\omega^3} + \\ + \frac{1}{4EI\omega^3}\left[P_2 u\left(x-c_2\right) + m_2 u_1\left(x-c_2\right)\right] + \\ + \frac{1}{4EI\omega^3}\left[P_3 u\left(x-c_3\right) + m_3 u_1\left(x-c_3\right)\right] \end{array}, \quad x \in [c_3, \ell] \\ 0 \ , \quad x \notin (0, \ell) \end{cases} \tag{6}$$

We note that in this relation of the deflection $\tilde{v}$ appear only two unknowns, namely: the reaction V_0 in O and the rotation of rod to the right in point O, $\tilde{v}'(0+0)$. These unknowns as well as the unknowns V_A, m_A, $\tilde{v}'(\ell-0)$ representing the reaction and moment in the A as well as the rotation of rod to the left in point A, respectively, will be determined from the following conditions:

$$\begin{array}{c} q\int\limits_0^\ell u\left(t\right)dt - V_0 u\left(\ell\right) + P_2 u\left(\ell-c_2\right) + P_3 u\left(\ell-c_3\right) + \\ + m_2 u_1\left(\ell-c_2\right) + m_3 u_1\left(\ell-c_3\right) + EI\tilde{v}'\left(0+0\right)u_2\left(\ell\right) = 0 \end{array} \tag{7}$$

$$\begin{array}{c} q\int\limits_0^\ell u_1\left(t\right)dt - V_0 u_1\left(\ell\right) + P_2 u_1\left(\ell-c_2\right) + P_3 u_1\left(\ell-c_3\right) + m_2 u_2\left(\ell-c_2\right) + \\ + m_3 u_2\left(\ell-c_3\right) + EI\left[\tilde{v}'\left(0+0\right)u_3\left(\ell\right) - 4\omega^3\tilde{v}'\left(\ell-0\right)\right] = 0 \end{array} \tag{8}$$

$$\begin{array}{c} q\int\limits_0^\ell u_2\left(t\right)dt - V_0 u_2\left(\ell\right) + P_2 u_2\left(\ell-c_2\right) + P_3 u_2\left(\ell-c_3\right) + \\ + m_2 u_3\left(\ell-c_2\right) + m_3 u_3\left(\ell-c_3\right) - 4EI\omega^3\tilde{v}'\left(0+0\right)u\left(\ell\right) = 0 \end{array} \tag{9}$$

$$\begin{array}{c} q\int\limits_0^\ell u_3\left(t\right)dt - V_0 u_3\left(\ell\right) + P_2 u_3\left(\ell-c_2\right) + P_3 u_3\left(\ell-c_3\right) - 4\omega^3 V_A - \\ -4\omega^4 m_2 u\left(\ell-c_2\right) - 4\omega^4 m_3 u\left(\ell-c_3\right) - 4EI\omega^4\tilde{v}'\left(0+0\right)u_1\left(\ell\right) = 0 \end{array} \tag{10}$$

The relations (7)-(10) were obtained from the condition that the support of the deflection should be $[0,\ \ell]$, namely $\mathrm{supp}\tilde{v} = [0, \ell]$.

From the above system of equations we shall obtain the unknowns V_0, V_A, $\tilde{v}'(\ell - 0)$ and $\tilde{v}'(0 + 0)$.

We have the expression

$$V_0 = \frac{4\omega^4 A_1 u\,(\ell) + A_3 u_2\,(\ell)}{4\omega^4 u^2\,(\ell) + u_2^2\,(\ell)} \tag{11}$$

$$\tilde{v}'\,(0+0) = \frac{A_3 u\,(\ell) + A_1 u_2\,(\ell)}{EI u_2^2\,(\ell) + 4\omega^4 EI u^2\,(\ell)} \tag{12}$$

$$\begin{aligned} V_A = {} & \tfrac{1}{4\omega^3} A_4 - \tfrac{1}{4\omega^3} \tfrac{A_3\left[4\omega^4 u_1(\ell)u(\ell) + u_3(\ell)u_2(\ell)\right]}{4\omega^4 u^2(\ell) + u_2^2(\ell)} - \\ & - \tfrac{1}{4\omega^3} \tfrac{4\omega^4 A_1\left[u_3(\ell)u(\ell) - u_1(\ell)u_2(\ell)\right]}{4\omega^4 u^2(\ell) + u_2^2(\ell)} \end{aligned} \tag{13}$$

$$\begin{aligned} \tilde{v}'\,(\ell-0) = {} & \tfrac{1}{4\omega^3 EI} A_2 - \tfrac{1}{4\omega^3 EI} \tfrac{A_1\left[4\omega^4 u(\ell)u_1(\ell) + u_2(\ell)u_3(\ell)\right]}{4\omega^4 u^2(\ell) + u_2^2(\ell)} - \\ & - \tfrac{1}{4\omega^3 EI} \tfrac{A_3\left[u_1(\ell)u_2(\ell) - u(\ell)u_3(\ell)\right]}{4\omega^4 u^2(\ell) + u_2^2(\ell)} \end{aligned} \tag{14}$$

where

$$\begin{aligned} A_1 = {} & q\int_0^\ell u\,(t)\,dt + P_2 u\,(\ell - c_2) + P_3 u\,(\ell - c_3) + \\ & + m_2 u_1\,(\ell - c_2) + m_3 u_1\,(\ell - c_3) \end{aligned} \tag{15}$$

$$\begin{aligned} A_2 = {} & q\int_0^\ell u_1\,(t)\,dt + P_2 u_1\,(\ell - c_2) + P_3 u_1\,(\ell - c_3) + \\ & + m_2 u_2\,(\ell - c_2) + m_3 u_2\,(\ell - c_3) \end{aligned} \tag{16}$$

$$\begin{aligned} A_3 = {} & q\int_0^\ell u_2\,(t)\,dt + P_2 u_2\,(\ell - c_2) + P_3 u_2\,(\ell - c_3) + \\ & + m_2 u_3\,(\ell - c_2) + m_3 u_3\,(\ell - c_3) \end{aligned} \tag{17}$$

$$\begin{aligned} A_4 = {} & q\int_0^\ell u_3\,(t)\,dt + P_2 u_3\,(\ell - c_2) + P_3 u_3\,(\ell - c_3) - \\ & - 4\omega^4 m_2 u\,(\ell - c_2) - 4\omega^4 m_3 u\,(\ell - c_3) \end{aligned} \tag{18}$$

3 Numerical Applications

We have considered an elastic rod with the following mechanical characteristics: flexural stiffness $EI = 2,1 \cdot 10^6\,\mathrm{Nm}^2$, the rigidity coefficient for the elastic foundation $k = 4,603 \cdot 10^7\,\frac{\mathrm{N}}{\mathrm{m}^2}$, the rod length $\ell = 8\,\mathrm{m}$, on the rod acts a uniformly distributed load $q = 1000\,\mathrm{N}$.

In Fig. 2 we have represented the deflection of the rod when we have two concentrated moments $P_2 = 100\,\mathrm{kN}$, $P_3 = 100\,\mathrm{kN}$, witch act on the points $c_2 = 1\,\mathrm{m}$, $c_3 = 7\,\mathrm{m}$ and $m_2 = m_3 = 0\,\mathrm{Nm}$. We observe that the deflection is symmetric with respect to the middle of the rod and in this neighborhood we have negative deflections.

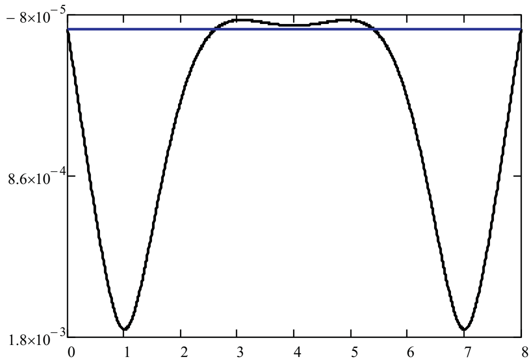

Fig. 2

Fig. 3 shows the deflection of the rod when we change the points of application of the concentrated loads, $c_2 = 3\,\mathrm{m}$, $c_3 = 5\,\mathrm{m}$. As we expected the graph of the deflection is symmetric with respect to the middle of the rod and at the ends of the rods we also have negative deflections.

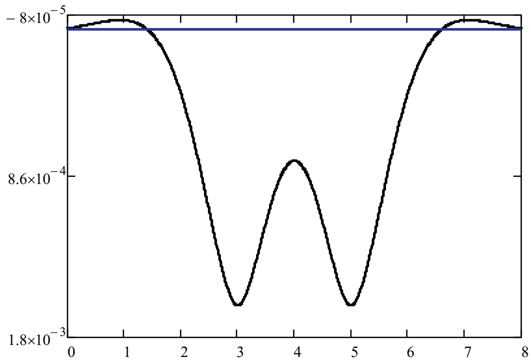

Fig. 3

In Fig. 4, we have represented the influence of the concentrated loads $P_2 = 100\,\mathrm{kN}$ and $P_3 = 80\,\mathrm{kN}$ on the deflection.

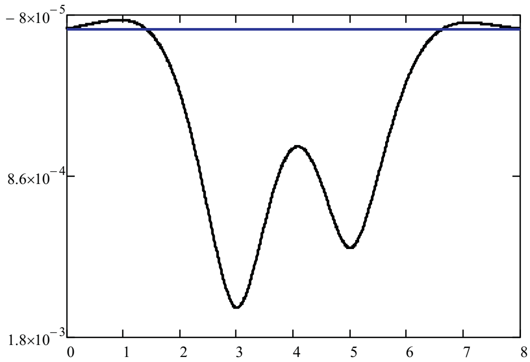

Fig. 4

In fig. 5 we considered $P_2 = P_3 = 0\,\mathrm{kN}$, and the concentrated moments $m_2 = 500\,\mathrm{Nm}$ and $m_3 = 500\,\mathrm{Nm}$.

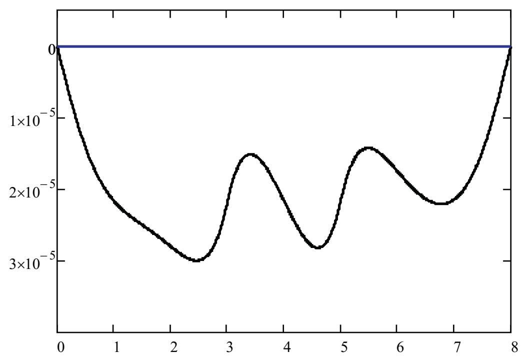

Fig. 5

In the last figure, we considered $P_2 = P_3 = 0\,\mathrm{kN}$, $m_2 = 500\,\mathrm{Nm}$ and $m_3 = -500\,\mathrm{Nm}$.

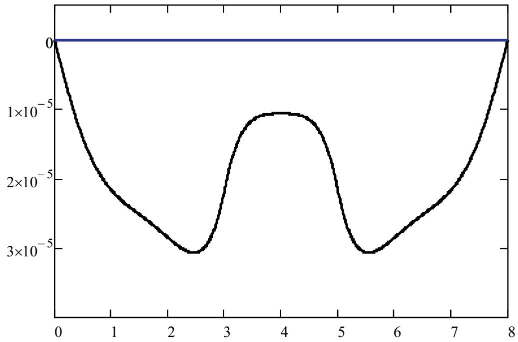

Fig. 6

We observe that in these cases we don't have negative deflections.

4 Conclusions

As it was pointed out in [2] the distribution theory represents the adequate framework to solve the boundary-value problems regarding the bending of the elastic rods on elastic foundation when we have external discontinuities (e.g. discontinuous loading) and internal discontinuities (e.g. owning to the mechanical properties). In this way the difference between continuous loads and discontinuous loads is vanish.

The obtained result allows a global analysis of the influence of each term: supports, the concentrated loads and the concentrated moments.

These influences are shown in several graphs.

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