Some investigations on a cosmological model

B. B. Chaturvedi, B. K. Gupta

Abstract. This paper contains the study of a six-dimensional space time, in which there are determined the scale factors $A(t)$, $B(t)$, $C(t)$, $D(t)$ and $E(t)$ with the help of the Einstein field equation and of the energy momentum tensor for the case of the perfect fluid. We have also evaluated some cosmological terms, like energy density ρ, energy pressure p, the Hubble parameter H, the deceleration parameter q, the anistropic mean A, and proved that all the parameters diverges at $t = -\frac{c_1}{nl}$, which emphasize this point as being a singular one. Plots for the comparative study of scale factors and cosmological terms are provided.

M.S.C. 2010: 85A40, 83F05.
Key words: Six-dimensional metric equation: Einstein field equation: Hubble parameter; deceleration parameter.

1 Introduction

For the description of the universe there exist numerous tools which are used by mathematicians and physicists. The cosmological models are such ones. The generally homogeneous and the anisotropic cosmological models are generally used. The Bianchi models I-IX describe the construction of homogeneous cosmology. Numerous works which modify the Einstein general relativity have been used by mathematicians and Physicists. The cosmological models, the Bianchi type model and the field equations have been studied by [2], Shrirams, and Cami et al.[6] by using different types of metric. In [1], Berman introduced a new model called FRW model, and obtained the solution of Einstein field equations by applying the law of variation of the Hubble parameter. The cosmological model with constant deceleration parameter has been studied by Johri and Desikan[3], Singh and Desikan[4], Maharaj and Naidu[5], Pradhan et al.[7] and Rahaman et al.[8], and Reddy et al.[10]. In [11], D. P. Teltumbade, J. K. Jumale K. D. Thengane gave the solution of the six-dimension static plane symmetric vacuum solutions in the $F(R)$ gravity. Friedman-Lemaitre-Robertson Walker ($FLRW$) studied the almost homogeneous and anisotropic space time. As consequence, in this paper, we consider a six-dimensional metric and evaluate all its scale factors and the cosmological terms. As well, we show that the scale factors vanish for $t = -\frac{c_1}{nl}$.

Applied Sciences, Vol.18, 2016, pp. 18-29.

2 The metric and field equations

We have considered the following six-dimensional metric.

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 - D^2d\mu^2 - E^2d\nu^2, \tag{2.1}$$

where A, B, C, D and E are functions of t only. The energy momentum tensor T_i^j for the perfect fluid is defined by

$$T_i^j = (\rho + p)V_iV^j - p\,g_i^j, \tag{2.2}$$

where p and ρ are the anistropic pressure and the energy density, respectively. We are taking V^i as the six-velocity of the particles, such that for $V^i = (0, 0, 0, 0, 0, 1)$, to have $V^iV_i = -1$. In 1915, for the first time Einstein published a paper that described how matter dynamically interacts with the geometry of space and time, which had impact in view of practical applications in GPS and information systems.

In 1916, Albert Einstein gave the equation

$$R_{i\,j} - \frac{1}{2}Rg_{i\,j} = kT_{i\,j}, \tag{2.3}$$

called *the Einstein field equation*, where R, $R_{i,j}$, k and T are respectively the scalar curvature, Ricci tensor, gravitational constant and energy momentum tensor.

Taking into consideration our model, the coefficients of the metric (2.1) form the matrix

$$K = \begin{pmatrix} -A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -D^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -E^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The determinant K is given by

$$|K| = -A^2\,B^2\,C^2\,D^2\,E^2,$$

and the non-vanishing Christoffel symbols corresponding to the metric are

$$\begin{gathered} \Gamma^6_{1\,1} = A_6A,\, \Gamma^6_{2\,2} = B_6B,\, \Gamma^6_{3\,3} = C_6C,\, \Gamma^6_{4\,4} = D_6D,\, \Gamma^6_{5\,5} = E_6E, \\ \Gamma^1_{6\,1} = \frac{A_6}{A},\, \Gamma^2_{6\,2} = \frac{B_6}{B},\, \Gamma^3_{6\,3} = \frac{C_6}{C},\, \Gamma^4_{6\,4} = \frac{D_6}{D},\, \Gamma^5_{6\,5} = \frac{E_6}{E}, \end{gathered} \tag{2.4}$$

where the index 6 denotes the derivative with respect to the time t, i.e., $A_6 = \frac{\partial A}{\partial t}$. We know that the Ricci tensor of type (0,2) is defined as

$$R_{i\,j} = \frac{\partial \Gamma^k_{k\,i}}{\partial x^j} - \frac{\partial \Gamma^k_{i\,j}}{\partial x^k} + \Gamma^q_{k\,i}\Gamma^k_{q\,j} - \Gamma^q_{i\,j}\Gamma^k_{q\,k}. \tag{2.5}$$

By using (2.4) in (2.5), we get all the non-zero components of the Ricci tensor,

$$
\begin{aligned}
R_{1\,1} &= -A\,A_{6\,6} - \tfrac{A\,A_6\,B_6}{A} - \tfrac{A\,A_6\,C_6}{C} - \tfrac{A\,A_6\,D_6}{D} - \tfrac{A\,A_6\,E_6}{E}.\\
R_{2\,2} &= -B\,B_{6\,6} - \tfrac{B\,A_6\,B_6}{A} - \tfrac{B\,B_6\,C_6}{C} - \tfrac{B\,B_6\,D_6}{D} - \tfrac{B\,B_6\,E_6}{E},\\
R_{3\,3} &= -C\,C_{6\,6} - \tfrac{C\,C_6\,A_6}{A} - \tfrac{C\,D_6\,C_6}{D} - \tfrac{C\,C_6\,B_6}{B} - \tfrac{C\,C_6\,E_6}{E},\\
R_{4\,4} &= -D\,D_{6\,6} - \tfrac{D\,A_6\,D_6}{A} - \tfrac{D\,B_6\,D_6}{B} - \tfrac{D\,C_6\,D_6}{C} - \tfrac{D\,D_6\,E_6}{E},\\
R_{5\,5} &= -E\,E_{6\,6} - \tfrac{E\,A_6\,E_6}{A} - \tfrac{E\,E_6\,B_6}{B} - \tfrac{E\,E_6\,D_6}{D} - \tfrac{E\,C_6\,E_6}{C},\\
R_{6\,6} &= \tfrac{A_{6\,6}}{A} + \tfrac{B_{6\,6}}{B} + \tfrac{C_{6\,6}}{C} + \tfrac{D_{6\,6}}{D} + \tfrac{E_{6\,6}}{E}.
\end{aligned}
\tag{2.6}
$$

Then, the scalar curvature tensor is given by

$$
R = R_{1\,1}\,g^{1\,1} + R_{2\,2}\,g^{2\,2} + R_{3\,3}\,g^{3\,3} + R_{4\,4}\,g^{4\,4} + R_{5\,5}\,g^{5\,5} + R_{6\,6}\,g^{6\,6},
\tag{2.7}
$$

and further, by using (2.6) in (2.7), we have

$$
\begin{aligned}
R =& 2\left(\frac{A_{6\,6}}{A} + \frac{B_{6\,6}}{B} + \frac{C_{6\,6}}{C} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E}\right)\\
&+ 2\left(\frac{A_6\,B_6}{A\,B} + \frac{A_6\,C_6}{A\,C} + \frac{A_6\,D_6}{A\,D} + \frac{A_6\,E_6}{A\,E} + \frac{C_6\,B_6}{C\,B}\right)\\
&+ \left(\frac{D_6\,B_6}{D\,B} + \frac{E_6\,B_6}{E\,B} + \frac{C_6\,D_6}{C\,D} + \frac{C_6\,E_6}{C\,E} + \frac{D_6\,E_6}{D\,E}\right).
\end{aligned}
\tag{2.8}
$$

This is the required scalar curvature in terms of scale factors. By the help of equation (2.2), (2.3), (2.6) and (2.8), we get field equations

$$
\begin{aligned}
&\tfrac{B_{6\,6}}{B} + \tfrac{C_{6\,6}}{C} + \tfrac{D_{6\,6}}{D} + \tfrac{E_{6\,6}}{E} + \tfrac{C_6\,B_6}{C\,B} + \tfrac{D_6\,B_6}{D\,B} + \tfrac{E_6\,B_6}{E\,B} + \tfrac{C_6\,D_6}{C\,D} + \tfrac{C_6\,E_6}{C\,E} + \tfrac{D_6\,E_6}{D\,E} = p.\\
&\tfrac{A_{6\,6}}{A} + \tfrac{C_{6\,6}}{C} + \tfrac{D_{6\,6}}{D} + \tfrac{E_{6\,6}}{E} + \tfrac{C_6\,A_6}{C\,A} + \tfrac{D_6\,A_6}{D\,A} + \tfrac{E_6\,A_6}{E\,A} + \tfrac{C_6\,D_6}{C\,D} + \tfrac{C_6\,E_6}{C\,E} + \tfrac{D_6\,E_6}{D\,E} = p,\\
&\tfrac{A_{6\,6}}{A} + \tfrac{B_{6\,6}}{B} + \tfrac{D_{6\,6}}{D} + \tfrac{E_{6\,6}}{E} + \tfrac{B_6\,A_6}{B\,A} + \tfrac{D_6\,A_6}{D\,A} + \tfrac{E_6\,A_6}{E\,A} + \tfrac{B_6\,D_6}{B\,D} + \tfrac{B_6\,E_6}{B\,E} + \tfrac{D_6\,E_6}{D\,E} = p,\\
&\tfrac{A_{6\,6}}{A} + \tfrac{B_{6\,6}}{B} + \tfrac{C_{6\,6}}{C} + \tfrac{E_{6\,6}}{E} + \tfrac{C_6\,E_6}{C\,E} + \tfrac{D_6\,B_6}{D\,B} + \tfrac{E_6\,A_6}{E\,A} + \tfrac{C_6\,A_6}{C\,A} + \tfrac{C_6\,B_6}{C\,B} + \tfrac{B_6\,E_6}{B\,E} = p,\\
&\tfrac{A_{6\,6}}{A} + \tfrac{B_{6\,6}}{B} + \tfrac{C_{6\,6}}{C} + \tfrac{D_{6\,6}}{D} + \tfrac{A_6\,B_6}{A\,B} + \tfrac{D_6\,B_6}{D\,B} + \tfrac{C_6\,B_6}{C\,B} + \tfrac{C_6\,A_6}{C\,A} + \tfrac{C_6\,E_6}{C\,E} + \tfrac{D_6\,A_6}{D\,A} = p,
\end{aligned}
\tag{2.9}
$$

and also

$$
\begin{aligned}
\rho = &-\left(\tfrac{A_6\,B_6}{A\,B} + \tfrac{A_6\,C_6}{A\,C} + \tfrac{A_6\,D_6}{A\,D} + \tfrac{A_6\,E_6}{A\,E} + \tfrac{C_6\,B_6}{C\,B}\right)\\
&+ \left(\tfrac{D_6\,B_6}{D\,B} + \tfrac{E_6\,B_6}{E\,B} + \tfrac{C_6\,D_6}{C\,D} + \tfrac{C_6\,E_6}{C\,E} + \tfrac{D_6\,E_6}{D\,E}\right).
\end{aligned}
$$

3 Solving the field equations

By recombining by subtraction the equations (2.9), we get

$$
\begin{aligned}
&\tfrac{A_{6\,6}}{A} - \tfrac{B_{6\,6}}{B} + \left(\tfrac{A_6}{A} - \tfrac{B_6}{B}\right)\left(\tfrac{C_6}{C} + \tfrac{D_6}{D} + \tfrac{E_6}{E}\right) = 0,\\
&\tfrac{B_{6\,6}}{B} - \tfrac{C_{6\,6}}{C} + \left(\tfrac{B_6}{B} - \tfrac{C_6}{C}\right)\left(\tfrac{A_6}{A} + \tfrac{D_6}{D} + \tfrac{E_6}{E}\right) = 0,\\
&\tfrac{C_{6\,6}}{C} - \tfrac{D_{6\,6}}{D} + \left(\tfrac{C_6}{C} - \tfrac{D_6}{D}\right)\left(\tfrac{A_6}{A} + \tfrac{B_6}{B} + \tfrac{E_6}{E}\right) = 0,\\
&\tfrac{D_{6\,6}}{D} - \tfrac{E_{6\,6}}{E} + \left(\tfrac{D_6}{D} - \tfrac{E_6}{E}\right)\left(\tfrac{C_6}{C} + \tfrac{B_6}{B} + \tfrac{A_6}{A}\right) = 0,\\
&\tfrac{E_{6\,6}}{E} - \tfrac{A_{6\,6}}{A} + \left(\tfrac{E_6}{E} - \tfrac{A_6}{A}\right)\left(\tfrac{C_6}{C} + \tfrac{D_6}{D} + \tfrac{B_6}{B}\right) = 0.
\end{aligned}
\tag{3.1}
$$

The first equation in (3.1) can be written as

$$\frac{\partial(\frac{A_6}{A}-\frac{B_6}{B})}{\partial t}+\left(\frac{A_6}{A}-\frac{B_6}{B}\right)\left(\frac{A_6}{A}+\frac{B_6}{B}+\frac{C_6}{C}+\frac{D_6}{D}+\frac{E_6}{E}\right)=0,$$

which by two-fold integration relative to t yields

$$\frac{A}{B}=d_1\,e^{k_1(\int\frac{1}{(ABCDE)}\,dt)}. \tag{3.2}$$

Similarly by rearranging and integrating the rest of the equations from (3.1), we infer

$$\tfrac{B}{C}=d_2\,e^{k_2(\int\frac{1}{(ABCDE)}\,dt)},\quad \tfrac{C}{D}=d_3\,e^{k_3(\int\frac{1}{(ABCDE)}\,dt)},$$
$$\tfrac{D}{E}=d_4\,e^{k_4(\int\frac{1}{(ABCDE)}\,dt)},\quad \tfrac{E}{A}=d_5\,e^{k_5(\int\frac{1}{(ABCDE)}\,dt)}.$$

We further propose the following result:

Theorem 3.1. *In the six dimensional space time, if* $k_1+k_2+k_3+k_4+k_5=0$*, then* $d_1\,d_2\,d_3\,d_4\,d_5=1$.

Proof. By multiplying the relations (3.2), we get

$$1=d_1\,d_2\,d_3\,d_4\,d_5\,e^{k_1+k_2+k_3+k_4+k_5\left(\int\frac{1}{(ABCDE)}\,dt\right)}.$$

Then, by using the relation $k_1+k_2+k_3+k_4+k_5=0$, we get $d_1\,d_2\,d_3\,d_4\,d_5=1$. □

By transvecting (2.2) by $g^{i\,k}$, we get

$$T^{j\,k}=(\rho+p)V^kV^j-p\,g^{j\,k},$$

and by taking the covariant derivative of $T^{j\,k}$ with respect to k, we get

$$T^{j\,k}_{;k}=\partial_k\,T^{j\,k}+T^{k\,\alpha}\,\Gamma^j_{\alpha\,k}+T^{j\,\alpha}\,\Gamma^k_{\alpha\,k},$$

which implies

$$T^{6\,6}_{;6}=\partial_6\,T^{6\,6}+T^{6\,\alpha}\,\Gamma^6_{\alpha\,6}+T^{6\,\alpha}\,\Gamma^6_{\alpha\,6},$$

and further, $T^{6\,6}_{;6}=\partial_6\,T^{6\,6}$. But $\partial_6\,T^{6\,6}=\rho_6$, and hence we get

$$T^{6\,6}_{;6}=\rho_6. \tag{3.3}$$

Similarly, we infer

$$\begin{aligned} &T^{6\,1}_{;1}=(\rho-p)\frac{A_6}{A},\quad T^{6\,2}_{;2}=(\rho-p)\frac{B_6}{B},\quad T^{6\,3}_{;3}=(\rho-p)\frac{C_6}{C},\\ &T^{6\,4}_{;4}=(\rho-p)\frac{D_6}{D}\quad \textit{and}\quad T^{6\,5}_{;5}=(\rho-p)\frac{E_6}{E}. \end{aligned} \tag{3.4}$$

We know that the energy conservation equation is given by $T^{j\,k}_{;k}=0$, which implies

$$T^{6\,6}_{;6}+T^{6\,1}_{;1}+T^{6\,2}_{;2}+T^{6\,3}_{;3}+T^{6\,4}_{;4}+T^{6\,5}_{;5}=0. \tag{3.5}$$

By using (3.3) and (3.4) in (3.5), we get

$$\rho_6 + (\rho - p)\left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right) = 0, \tag{3.6}$$

which is the required energy conservation equation in terms of scale factors.

Now we propose the following:

Theorem 3.2. *In a six dimensional space time the energy density ρ is constant if and only if either $\rho = p$, or $ABCDE = constant$.*

Proof. If we take $\rho = costant$, then $\rho_6 = 0$. Replacing this in (3.6), we get

$$(\rho - p)\left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right) = 0,$$

which implies either $\rho = p$ or

$$\left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right) = 0.$$

This infers that either $\rho = p$ or $ABCDE = constant$. Conversely, let $\rho = p$. Then from (3.6) we infer $\rho_6 = 0$, which implies $\rho = constant$; if we take $ABCDE = constant$, then we can write

$$\log A + \log B + \log C + \log D + \log E = \log(constant).$$

Differentiating this with respect to t, we get

$$\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E} = 0,$$

and using this in (3.6), we get $\rho_6 = 0$. □

4 The scale factor

We have seven unknowns A, B, C, D, E, p and ρ in five equations (3.1). Therefore, after solving these equations, we can not find an explicit solution. To solve this issue, we consider other two parameters related to these unknowns. In [9], Pradhan and Chouhan consider the relation $A = (BC)^m$, where m is positive constant, and obtained the explicit solution of the Einstein field equations of Bianchi type-I. In [1], Berman gave the special law of variation for the Hubble parameter which has a constant value of deceleration parameter. This parameter is defined by $H = l(ABC)^{\frac{-n}{3}}$, where $l > 0$ and $n \geq 0$.

In the following we consider

$$A = (BCDE)^m, \tag{4.1}$$

and

$$H = l(ABCDE)^{\frac{-n}{5}}, \tag{4.2}$$

we also define the average scale factor R for the six-dimensional metric equation, by the relation $R^5 = ABCDE$. We know that the generalized Hubble parameter H for six-dimensions is given by

$$H = \frac{R_6}{R} = \frac{1}{5}\left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right), \tag{4.3}$$

which, replaced in (4.2), leads to

$$l(ABCDE)^{\frac{-n}{5}} = \frac{1}{5}\left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right),$$

which, integrated with respect to t, infers

$$(ABCDE) = (nlt + c_1)^{\frac{5}{n}}, \tag{4.4}$$

for $n \neq 0$, and

$$(ABCDE) = c_2^5\, e^{5lt}, \tag{4.5}$$

for $n = 0$, where c_1 and c_2 are intergration constants. Thus (4.4) and (4.5) show the power expansion and exponential expansion respectively. In [9], Pradhan and Chouhan obtained scale factors by using the power law and the exponential law in anisotropic Bianchi type-I model string cosmology. By solving equations (3.2), (4.1) and (4.4), we get

$$A(t) = (nlt + c_1)^{\frac{5m}{n(m+1)}}, \tag{4.6}$$

and using (4.6) in (3.2), we get

$$B(t) = (nlt + c_1)^{\frac{5m}{n(m+1)}} d_1^{-1} e^{-\frac{k_1}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}}, \qquad n \neq 5. \tag{4.7}$$

Similarly, for $n \neq 5$, we obtain

$$\begin{aligned} C(t) &= (nlt + c_1)^{\frac{5m}{n(m+1)}} d_1^{-1} d_2^{-1} e^{-\frac{k_1+k_2}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}}, \\ D(t) &= (nlt + c_1)^{\frac{5m}{n(m+1)}} d_1^{-1} d_2^{-1} d_3^{-1} e^{-\frac{k_1+k_2+k_3}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}}, \\ E(t) &= (nlt + c_1)^{\frac{5m}{n(m+1)}} d_1^{-1} d_2^{-1} d_3^{-1} d_4^{-1} e^{-\frac{k_1+k_2+k_3+k_4}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}}. \end{aligned} \tag{4.8}$$

Hence equation (2.1) reduces to

$$\begin{aligned} ds^2 = \;& dt^2 - [(nlt + c_1)^{\frac{5m}{n(m+1)}}]^2 \Big(dx^2 + d_1^{-2} e^{-2\frac{k_1}{l(n-5)}(nlt+c_1)^{\frac{n-5}{5}}} dy^2 \\ & + d_1^{-2} d_2^{-2} e^{-2\frac{k_1+k_2}{l(n-5)}(nlt+c_1)^{\frac{n-5}{5}}} dz^2 + d_1^{-2} d_2^{-2} d_3^{-2} e^{-2\frac{k_1+k_2+k_3}{l(n-5)}(nlt+c_1)^{\frac{n-5}{5}}} d\mu^2 \\ & + d_1^{-2} d_2^{-2} d_3^{-2} d_4^{-2} e^{-2\frac{k_1 k_2+k_3+k_4}{l(n-5)}(nlt+c_1)^{\frac{n-5}{5}}} d\nu^2\Big). \end{aligned}$$

Taking two times the covariant derivative of (4.6) with respect to t, we get

$$A_{6\,6} = \frac{5ml^2(5m - n(m+1))}{(m+1)^2}(nlt + c_1)^{\frac{5m}{n(m+1)}-2}. \tag{4.9}$$

From (4.6) and (4.9), we have

$$\frac{A_{66}}{A} = \frac{5ml^2(5m - n(m+1))}{(m+1)^2}(nlt + c_1)^{-2}.$$

Taking once the covariant derivative of (4.7) with respect to t, we get

$$\begin{aligned} B_6 =& \frac{5ml}{(m+1)}(nlt + c_1)^{\frac{5m}{n(m+1)}-1} d_1^{-1} e^{-\frac{k_1}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}} \\ &- (nlt + c_1)^{\frac{5m}{n(m+1)}} d_1^{-1} k_1 (nlt + c_1)^{\frac{-5}{n}} e^{-\frac{k_1}{l(n-5)}(nlt+c_1)^{\frac{n-5}{n}}}. \end{aligned} \tag{4.10}$$

From (4.7) and (4.10), we infer

$$\frac{B_6}{B} = \frac{5ml}{(m+1)}(nlt + c_1)^{-1} - k_1(nlt + c_1)^{\frac{-5}{n}}. \tag{4.11}$$

Similarly, we obtain

$$\begin{aligned} \tfrac{A_6}{A} &= \tfrac{5ml}{(m+1)}(nlt + c_1)^{-1}, \\ \tfrac{C_6}{C} &= \tfrac{5ml}{(m+1)}(nlt + c_1)^{-1} - (k_1 + k_2)(nlt + c_1)^{\frac{-5}{n}}, \\ \tfrac{D_6}{D} &= \tfrac{5ml}{(m+1)}(nlt + c_1)^{-1} - (k_1 + k_2 + k_3)(nlt + c_1)^{\frac{-5}{n}}, \\ \tfrac{E_6}{E} &= \tfrac{5ml}{(m+1)}(nlt + c_1)^{-1} - (k_1 + k_2 + k_3 + k_4)(nlt + c_1)^{\frac{-5}{n}}. \end{aligned} \tag{4.12}$$

Again, taking the covariant derivative of (4.10) and dividing by (4.7), it follows that

$$\begin{aligned} \frac{B_{66}}{B} =& \frac{5ml^2(5m - n(m+1))}{(m+1)^2}(nlt + c_1)^{-2} \\ &- \left(\frac{5lk_1(2m-1)}{m+1}\right)(nlt + c_1)^{\frac{-n-5}{n}} + k_1^2(nlt + c_1)^{\frac{-10}{n}}. \end{aligned} \tag{4.13}$$

Similarly, we find

$$\begin{aligned} \tfrac{C_{66}}{C} =& \tfrac{5ml^2(5m-n(m+1))}{(m+1)^2}(nlt + c_1)^{-2} \\ &- \left(\tfrac{5l(k_1+k_2)(2m-1)}{m+1}\right)(nlt + c_1)^{\frac{-n-5}{n}} + (k_1 + k_2)^2(nlt + c_1)^{\frac{-10}{n}}, \\ \tfrac{D_{66}}{D} =& \tfrac{5ml^2(5m-n(m+1))}{(m+1)^2}(nlt + c_1)^{-2} \\ &- \left(\tfrac{5l(k_1+k_2+k_3)(2m-1)}{m+1}\right)(nlt + c_1)^{\frac{-n-5}{n}} \\ &+(k_1 + k_2 + k_3)^2(nlt + c_1)^{\frac{-10}{n}}, \\ \tfrac{E_{66}}{E} =& \tfrac{5ml^2(5m-n(m+1))}{(m+1)^2}(nlt + c_1)^{-2} \\ &- \left(\tfrac{5l(k_1+k_2+k_3+k_4)(2m-1)}{m+1}\right)(nlt + c_1)^{\frac{-n-5}{n}} \\ &+(k_1 + k_2 + k_3 + k_4)^2(nlt + c_1)^{\frac{-10}{n}}, \end{aligned} \tag{4.14}$$

Using (4.11), (4.12), (4.13), and (4.14) in (2.9), we get

$$\begin{aligned} p =& \tfrac{275(ml)^2 - 25nml^2(m+1)}{(m+1)^2}(nlt + c_1)^{-2} - \\ &- \tfrac{(5lL_1)(5m-1)}{m+1}(nlt + c_1)^{\frac{(-n-5)}{n}} + L_2(nlt + c_1)^{\frac{-10}{n}}, \end{aligned} \tag{4.15}$$

where $L_1 = 4k_1 + 3k_2 + 2k_3 + k_4$, and

$$L_2 = 10k_1^2 + 5k_2^2 + 3k_3^2 + k_4^2 + 16k_1k_2 + 10k_1k_3 + 5k_1k_4 + 8k_3k_2 + 4k_2k_4 + 3k_3k_4.$$

Similarly, using (4.11) and (4.12), we get

$$\begin{aligned}\rho = \quad & -\tfrac{250(ml)^2}{(m+1)^2}(nlt+c_1)^{-2} + \\ & \tfrac{20lmL_1}{(m+1)}(nlt+c_1)^{\frac{(-n-5)}{n}} - L_3(nlt+c_1)^{\frac{-10}{n}},\end{aligned} \tag{4.16}$$

where $L_3 = 6k_1^2 + 3k_2^2 + k_3^2 + 8k_1k_2 + 6k_1k_3 + 3k_1k_4 + 4k_3k_2 + 2k_2k_4 + k_3k_4$.

From equation (4.16) we see that if $\rho \geq 0$ then

$$t \leq \frac{1}{nl}\left(\left(\frac{250(ml)^2}{(m+1)^2(\frac{5lmL_3}{(m+1)}(nlt+c_1)^{\frac{(-n-5)}{n}} - L_4(nlt+c_1)^{\frac{-10}{n}})}\right)^{\frac{1}{2}} - c_1\right).$$

From (4.16), we can say that all the parameters are diverging at $t = -\frac{c_1}{nl}$; this shows that the model has a singularity at $t = -\frac{c_1}{nl}$, and implies that $t = 0$ if only if $c_1 = 0$. Since the pressure p and the energy density ρ are diverging at ($t = -\frac{c_1}{nl}$), this type of singularity is called *point type singularity*.

The rates of expansion H_i along X, Y, Z, μ and ν are

$$\begin{aligned} H_x &= \tfrac{A_6}{A} = \tfrac{5ml}{(m+1)}(nlt+c_1)^{-1}, \\ H_y &= \tfrac{B_6}{B} = \tfrac{5ml}{(m+1)}(nlt+c_1)^{-1} - k_1(nlt+c_1)^{\frac{-5}{n}}, \\ H_z &= \tfrac{C_6}{C} = \tfrac{5ml}{(m+1)}(nlt+c_1)^{-1} - (k_1+k_2)(nlt+c_1)^{\frac{-5}{n}}, \\ H_\mu &= \tfrac{D_6}{D} = \tfrac{5ml}{(m+1)}(nlt+c_1)^{-1} - (k_1+k_2+k_3)(nlt+c_1)^{\frac{-5}{n}}, \\ H_\nu &= \tfrac{E_6}{E} = \tfrac{5ml}{(m+1)}(nlt+c_1)^{-1} - (k_1+k_2+k_3+k_4)(nlt+c_1)^{\frac{-5}{n}}. \end{aligned} \tag{4.17}$$

Using (4.17) in (4.3), we infer

$$H = \frac{5ml}{(m+1)}(nlt+c_1)^{-1} - \frac{1}{5}L_1(nlt+c_1)^{\frac{-5}{n}}. \tag{4.18}$$

We know that the description of homogeneous universe is given by the values of all density parameters and the present Hubble parameter H, which we need to compute. As well, there are some important physical quantities of observational interest in cosmology: the expansion scalar θ, the Hubble parameter H and the deceleration parameter q, defined as follows.

The *expansion scalar* θ is the quantity given by

$$\theta = v^i_{;i} = \left(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\right).$$

By using (4.11)-(4.12) in this expression, we have

$$\theta = \frac{25ml}{(m+1)}(nlt+c_1)^{-1} - L_1(nlt+c_1)^{\frac{-5}{n}}. \tag{4.19}$$

The *deceleration parameter* is defined by

$$q = \frac{\partial}{\partial t}\left(\frac{1}{H}\right) - 1,$$

and by using (4.18), we get

$$q = \left(\frac{\frac{5ml^2 n}{m+1}(nlt+c_1)^{-2} - L_5 l(nlt+c_1)^{\frac{-5-n}{n}}}{(\frac{5ml}{(m+1)}(nlt+c_1)^{-1} - \frac{L_5}{5}(nlt+c_1)^{\frac{-5}{n}})^2} \right), \tag{4.20}$$

The expression of the *mean anistropy* is

$$A = \frac{1}{n-1}\sum_{i=1}^{n-1}\left(\frac{\Delta H_i}{H}\right)^2,$$

where $\Delta H_i = H_i - H$; by using (4.17) and (4.18), we infer

$$A = \left(\frac{\left(\frac{4ml}{m+1}(nlt+c_1)^{-1} - \frac{4}{25}L_1(nlt+c_1)^{-\frac{5}{n}}\right)}{\left(\frac{5ml}{m+1}(nlt+c_1)^{-1} - \frac{1}{5}L_1(nlt+c_1)^{-\frac{5}{n}}\right)} \right)^2. \tag{4.21}$$

From (4.15), (4.16), (4.18), (4.19), (4.20) and (4.21), we infer the following:

Theorem 4.1. *In the 6-dimensional space time, the cosmological terms of energy density ρ, pressure p, Hubble parameter H, expansion scalar θ, deceleration parameter q and anistropic mean A diverge at $t = -\frac{c_1}{nl}$. In other words, we can say that the point $t = -\frac{c_1}{nl}$ is a singular point for these cosmological terms.*

5 Plots and conclusions

All the scale factors were explicitly obtained in (4.6)-(4.8). As well, the cosmological terms of energy pressure p, energy density ρ, Hubble parameter H, expansion scalar θ, deceleration parameter q and anistropic mean were evaluated and given in (4.15), (4.16), (4.18), (4.19), (4.20) and (4.21) respectively. From these equations we remark that all cosmological terms diverge at $t = -\frac{c_1}{nl}$ and the scale factors vanish at $t = -\frac{c_1}{nl}$. Therefore we can say that $t = -\frac{c_1}{nl}$ is a singular point for all cosmological terms. The t-plots of these quantities are given in Fig.1, Fig.2, Fig.3, Fig.4, Fig.5 and Fig.6.

From Fig.2 one can easily see that the cosmological evolution of the six-dimensional space time is expansionary, with all the five scale factors monotonically increasing functions of time t. Also, the variation of all the scale factors relative to time, are shown in Fig.2, and we can see that $A(t)$ maximally varies in terms of T, while $E(t)$ minimally varies. From Fig.1 and fig.4, it is clear that the energy density and the energy pressure are both independent of t in the six-dimensional cosmological evolution of model. As well, Fig.5 shows the variation of the expansion scalar θ with the cosmic time t, and reveals that the expansion scalar θ tends to zero as t approaches infinity. From Fig.3. and Fig.6. one notices the variation of the deceleration parameter q and the Hubble parameter H in terms of t.

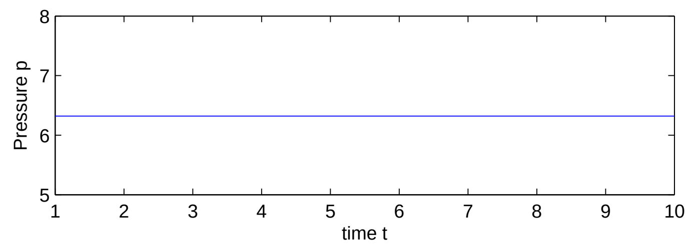

Figure 1: Pressure vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

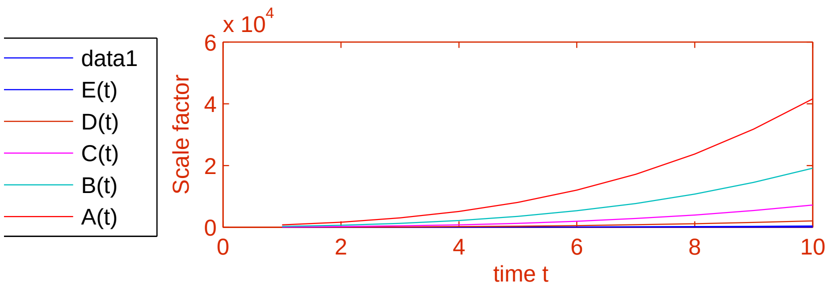

Figure 2: Scale factor vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

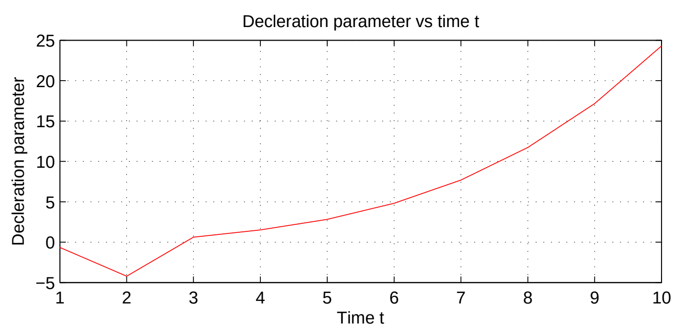

Figure 3: Deceleration parameter vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

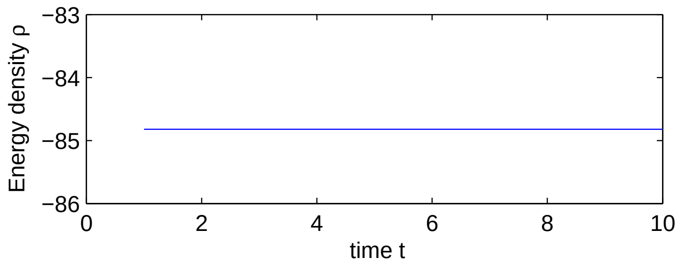

Figure 4: Energy density vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

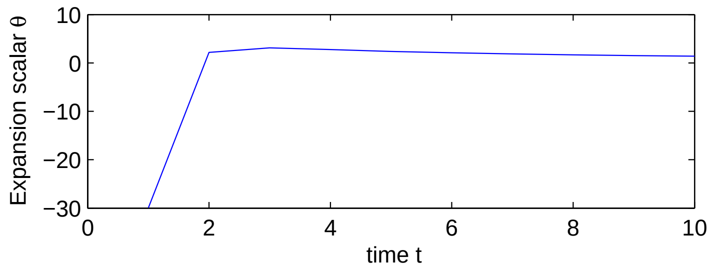

Figure 5: Expansion scalar vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

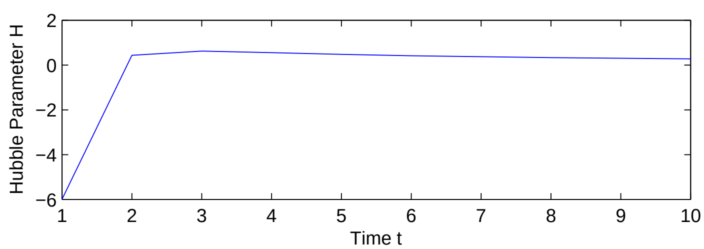

Figure 6: The Hubble parameter vs. time t, where $(l = m = n = c = 0.5)$ and $(k_1, k_2, k_3, k_4, k_5) = (0.1, 0.2, 0.3, 0.4, 0.5)$

Acknowledgements. The second author expresses his thanks to (UGC) New Delhi, India for providing Junior Research Fellowship (JRF).

References

[1] M.S. Berman, *A special law of variation of Hubble parameter*, Nuovo Cimento 74 (1983), 182-186.

[2] J. Hajj-Boutros, *On hypersurface homogeneous space-times*, J. Math. Phys. 26 (1985), 2297.

[3] V. B.Johri, K. Desikan, *Cosmological models with constant deceleration parameter in Nordtvedt's theory*, Pramana J. Phys. 21, 7 (1989), 697-704.

[4] G.P.Singh, K. Desikan *A new class of cosmological models in Lyra geometry* Int. J. Theor. Phys. 29 (1990), 901.

[5] S. D. Maharaj, R. Naidoo, *Solution of the field equations and the deceleration parameter*, Astrophys. Space Sci. 208 (1993), 361.

[6] U. Camci, I. Yavuz, H. Baysal, I. Tarhan, I. Yilmaz, *Generation of Bianchi type V universes filled with a perfect fluid* , Astrophys. Space Sci. 275 (2001), 391.

[7] A. Pradhan, *Some magnetized bulk viscous string cosmological models in cylindrically symmetric inhomogeneous universe with variable Λ-term* , Communications in Theor. Phys. 51 (2002), 367.

[8] F. Rahman, N. Begum and B. C. Bhui, *Cosmological models with negative constant deceleration parameter in Lyra geometry*, Astrophys. Space Sci. 299 (2004), 211.

[9] A. Pradhan, D. S. Chouhan, *Anisotropic Bianchi type-I models in string cosmology*, Astrophys. Space Sci. 331 (2006), 697-704.

[10] D. R. K. Reddy, R. M. V. Subha and R. G. Koteswara, *A cosmological model with negative constant deceleration parameter in a scalar tensor theory*, Astrophys. Space Sci. 306 (2006),171.

[11] D. P. Teltumbade, J. K. Jumale and K. D. Thengane, *Six-dimensional static plane symmetric vacuum solutions in $f(R)$ gravity*, Prespacetime J. 3, 5 (2012), 495-503.

Authors' address:

Braj Bhushan Chaturvedi *and* Brijesh Kumar Gupta
Department of Pure and Applied Mathematics,
Guru Ghasidas Vishwavidyalaya Bilaspur (Chhattisgarh), India.
E-mail: brajbhushan25@gmail.com & brijeshggv75@gmail.com