COMMENTS ON THE PAPER BY M.BOUNKHEL

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All the results obtained in the paper: On Minimizing the norm of linear maps in C_p-classes, by M.Bounkhel [1] are already known and published in [2] and [3]. To be very frank, this paper is very misleading and the so-called results obtained in this paper are also represent a distortion of the known facts.

1. The paper is the same paper as [3], where he changed only the title. The title of the paper is misleading.

2. The reference of Keckić which is the essence of the paper is incomplete, it is written to (appear) but this paper is already appeared see [4]. It is clear that the author have not seen the paper of Keckić in all. Since he wrote (to appear), this means that the author have a copy of the paper before publication (you have to ask him to send you a copy of this paper, I am sure he has not a copy). How he has a copy of the paper and he wrote a wrong definition of the φ-derictionnal derivative (which is φ-Gateaux derivative as in [4] and [5] which is not mentioned in all in Bounkhel paper Please compare the definition of Bounkhel and the follwing correct definition given by Keckić

Definition 0.1. [5] *Let $(\mathcal{B}, \|\cdot\|)$ be an arbitrary Banach space, $x, y \in \mathcal{B}$, φ in $[0, 2\pi)$, and $F : \mathcal{B} \to \mathbb{R}$. We define the φ-Gâteaux derivative of F at a vector $x \in \mathcal{B}$, in $y \in \mathcal{B}$ and φ direction by*

$$D_\varphi F(x; y) = \lim_{t \to 0^+} \frac{F(x + te^{i\varphi} y) - F(x)}{t}.$$

From the Clarckson-McCarthy inequalities it follows that the dual space $C_p^* \cong C_q$ is strictly convex. From this we can derive that every non zero point in C_p is a smooth point of the corresponding sphere. So we can check what is the unique support functional.

However, if the dual space is not strictly convex, there are many points which are not smooth. For instance, it happens in C_1, C_∞ and $B(H)$. The concept of φ-Gateaux derivative will be used in order to substitute the usual concept of Gateaux derivative at points which are not smooth. It is clear that the Gateaux derivative of Cp-classes always exists and we don't need in all φ-directionnel derivative see ([2])

5. There are many fatal erros in the paper (they could not made by a specialist in the field). For example:

a) Page 40 Introduction line 9-10 . The author wrote: The operator T is said to be belong to the Schatten p-classes C_p if

$$\|T\|_p = \left[\sum_{i=1}^{\infty} s_i(T)^p\right]^{\frac{1}{p}} = [tr|T|^p]^{\frac{1}{p}}, \quad 1 \leq p < \infty,$$

Which is not true he should add at the end of the formulas $< \infty$.

b) Page 42 Remark 3.1 line 10. The statement $DF(x,y) = ||y||$ is incorrect. For example, $DF(x,-x) = -||x||$. Indeed, in Hilbert spaces one has $DF(x,y) = \frac{Re\langle x,y\rangle}{||x||}$.

c) Page 44 . 2. is a spcial case of 1. Why he wrote, the operator $\sum_1^n A_iXB_i - X$ which is clearly a particular case of the operator $\sum_1^n A_iXB_i$. See also page 41. Line 16, 17.

7. This clearly shows that the author copied all the results from that paper. By this all the results and other definitions are wrong in one way or the other.

References

[1] M.Bounkhel, On Minimizing the norm of linear maps in Cp-classes, Applied Sciences, Vol 8, 2006, 40-47.

[2] Gateaux Derivative and orthogonality in C_p- classes, J.Ineq. Pure Appl. Math (JIPAM), Volume 7, Issue 2, Article 77, 2006.

[3] Global minimum and Orthogonality in C_p- classes, Math Nachr (Ms-Nr. 1019/77-03), to appear.

[4] D. Keckić, Orthogonality of the range and the kernel of some elementary operators, Proc. Amer. Math. Soc. 128 -11(2000), 3369-3377.

[5] D. Keckić, Orthogonality in $\mathcal{S}_1$ and $\mathcal{S}_\infty$., Jour. Op. Th. 51(2004) no1, 89-104.

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