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# ON THE RECURSIVE SYSTEM $x_{n+1} = A + \frac{x_{n-m}}{y_n}$, $y_{n+1} = B + \frac{y_{n-m}}{x_n}$

QIANHONG ZHANG, LIHUI YANG AND JINGZHONG LIU

Abstract. In this paper, we investigate the boundedness, persistence and global asymptotic stability of positive solutions of the system of two nonlinear difference equations

$$x_{n+1} = A + \frac{x_{n-m}}{y_n}, \qquad y_{n+1} = B + \frac{y_{n-m}}{x_n}, \qquad n = 0, 1, \cdots,$$

where $A, B \in (0, \infty)$, $x_i \in (0, \infty)$, $y_i \in (0, \infty)$, $i = -m, -m+1, \ldots, 0$.

## 1. Introduction

The study of dynamical behavior of various nonlinear differences is not only of interest in their own right, but the results can help establish the general theory of nonlinear difference equations. Amleha, Grovea, Ladasa, et al. [**1**] investigated the global stability, the boundedness character and the periodic nature of the difference equation

$$x_{n+1} = \alpha + \frac{x_{n-1}}{x_n}, \quad n = 0, 1, \ldots,$$

where $x_{-1}, x_0 \in R$ and $\alpha > 0$.

Elowaidy, Ahmed and Mousa [**2**] investigated local stability, oscillation and boundedness character of the difference equation

$$x_{n+1} = \alpha + \frac{x_{n-1}^p}{x_n^p}, \quad n = 0, 1, \ldots,$$

where $\alpha, p \in (0, +\infty)$. Also Stevic [**3**] studied dynamical behavior of this difference equation. Other related difference equation readers can refer to references [**4**]–[**10**].

In recent years, nonlinear difference equation systems have attracted considerable interest [**11**]–[**18**]. In particular, Papaschinopoulos and Papadopoulos [**13**] studied the dynamics of the system of rational difference equations

$$x_{n+1} = A + \frac{x_n}{y_{n-m}}, \quad y_{n+1} = B + \frac{y_n}{x_{n-m}}, \quad n = 0, 1, \ldots \tag{1.1}$$

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for the case (I) $A > 1$ and $B > 1$, and (II) $A < 1$ and $B < 1$; while Camouzis and Papaschinopoulos [**14**] studied system (1.1) for the case $A = B = 1$.

Papaschinopoulos and Schinas [**15**] studied the system of two nonlinear difference equations

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \qquad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \qquad n = 0, 1, \cdots, \tag{1.2}$$

where $p$, $q$ are positive integers and $A > 0$.

Zhang and Yang, Evans and Zhu [**18**] investigated the system of rational difference equations

$$x_{n+1} = A + \frac{y_{n-m}}{x_n}, \qquad y_{n+1} = A + \frac{x_{n-m}}{y_n}, \qquad n = 0, 1, \ldots, \tag{1.3}$$

where $A \in (0, \infty)$ and the initial conditions $x_i \in (0, \infty)$, $y_i \in (0, \infty)$, $i = -m, -m+1, \ldots, 0$, are arbitrary nonnegative numbers.

Our aim in this paper is to investigate the boundedness, persistence and global asymptotic stability of positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{x_{n-m}}{y_n}, \qquad y_{n+1} = B + \frac{y_{n-m}}{x_n}, \qquad n = 0, 1, \ldots, \tag{1.4}$$

where $A, B \in (0, \infty)$, and initial conditions $x_i, y_i \in (0, \infty)$, $i = -m, -m+1, \ldots, 0$.

## 2. The case $A < 1$ and $B < 1$

In this section, we are concerned with the asymptotic behavior of positive solution of (1.4) for case $A < 1$, $B < 1$.

**Theorem 2.1.** *Suppose that $0 < A < 1$, $0 < B < 1$. Let $\{(x_n, y_n)\}$ be an arbitrary positive solution of* (1.4). *The following statements are true:*

(i) *If $m$ is odd and $0 < x_{2k-1} < 1$, $0 < y_{2k-1} < 1$, $x_{2k} > \frac{1}{1-B}$, $y_{2k} > \frac{1}{1-A}$ for $k = \frac{1-m}{2}, \frac{3-m}{2}, \ldots, 0$, then*

$$\lim_{n\to\infty} x_{2n} = \infty, \quad \lim_{n\to\infty} y_{2n} = \infty, \quad \lim_{n\to\infty} x_{2n+1} = A, \quad \lim_{n\to\infty} y_{2n+1} = B.$$

(ii) *If $m$ is odd and $0 < x_{2k} < 1$, $0 < y_{2k} < 1$, $x_{2k-1} > \frac{1}{1-B}$, $y_{2k-1} > \frac{1}{1-A}$ for $k = \frac{1-m}{2}, \frac{3-m}{2}, \ldots, 0$, then*

$$\lim_{n\to\infty} x_{2n} = A, \quad \lim_{n\to\infty} y_{2n} = B, \quad \lim_{n\to\infty} x_{2n+1} = \infty, \quad \lim_{n\to\infty} y_{2n+1} = \infty.$$

(iii) *If $m$ is even and $0 < x_{2k-1} < 1$, $y_{2k-1} > \frac{1}{1-A}$, $x_{2k} > \frac{1}{1-B}$, $0 < y_{2k} < 1$ for $k = \frac{2-m}{2}, \frac{4-m}{2}, \ldots, 0$ and $x_{-m} > \frac{1}{1-B}$, $0 < y_{-m} < 1$, then*

$$\lim_{n\to\infty} x_{2n} = A, \quad \lim_{n\to\infty} y_{2n} = \infty, \quad \lim_{n\to\infty} x_{2n+1} = \infty, \quad \lim_{n\to\infty} y_{2n+1} = B.$$

(iv) *If $m$ is even and $0 < x_{2k} < 1, y_{2k} > \frac{1}{1-A}$, $x_{2k-1} > \frac{1}{1-B}$, $0 < y_{2k-1} < 1$ for $k = \frac{2-m}{2}, \frac{4-m}{2}, \ldots, 0$ and $0 < x_{-m} < 1$, $y_{-m} > \frac{1}{1-A}$, then*

$$\lim_{n\to\infty} x_{2n} = \infty, \quad \lim_{n\to\infty} y_{2n} = B, \quad \lim_{n\to\infty} x_{2n+1} = A, \quad \lim_{n\to\infty} y_{2n+1} = \infty.$$

*Proof.* (i) It is clear that

$$0 < x_1 = A + \frac{x_{-m}}{y_0} < A + \frac{1}{y_0} < A + (1 - A) = 1,$$
$$0 < y_1 = B + \frac{y_{-m}}{x_0} < B + \frac{1}{x_0} = B + (1 - B) = 1.$$
$$x_2 = A + \frac{x_{1-m}}{y_1} > x_{1-m} > \frac{1}{1 - B},$$
$$y_2 = B + \frac{y_{1-m}}{x_1} > y_{1-m} > \frac{1}{1 - A}.$$

By induction for $n = 1, 2, \dots,$ we have

$$0 < x_{2n-1} < 1, \quad 0 < y_{2n-1} < 1, \quad x_{2n} > \frac{1}{1 - B}, \quad y_{2n} > \frac{1}{1 - A}. \tag{2.1}$$

So, for $n \geq (m + 2)/2$.

$$x_{2n} = A + \frac{x_{2n-(m+1)}}{y_{2n-1}} > A + x_{2n-(m+1)} = 2A + \frac{x_{2n-(2m+2)}}{y_{2n-(m+2)}} > 2A + x_{2n-(2m+2)},$$
$$y_{2n} = B + \frac{y_{2n-(m+1)}}{x_{2n-1}} > B + y_{2n-(m+1)} = 2B + \frac{y_{2n-(2m+2)}}{x_{2n-(m+2)}} > 2B + y_{2n-(2m+2)},$$

from which we obtain $\lim_{n\to\infty} x_{2n} = \infty$, $\lim_{n\to\infty} y_{2n} = \infty$. Noting (2.1) and taking limits on both sides of the system

$$x_{2n+1} = A + \frac{x_{2n-m}}{y_{2n}}, \qquad y_{2n+1} = B + \frac{y_{2n-m}}{x_{2n}},$$

we obtain $\lim_{n\to\infty} x_{2n+1} = A$, $\lim_{n\to\infty} y_{2n+1} = B$.

The proof of affirmations (ii), (iii) and (iv) are similar, we omit it. □

## 3. The Case $A = 1$ and $B = 1$

In this section, we discuss the boundedness and persistence of positive solutions to system (1.4) for the case $A = 1, B = 1$.

**Theorem 3.1.** *Suppose that $A = B = 1$. Then every positive solution of system* (1.4) *is bounded and persists.*

*Proof.* The proof of Theorem 3.1 is similar to [**18**]. Let $\{(x_n, y_n)\}$ be a positive solution of (1.4). Clearly, $x_n > 1$, $y_n > 1$ for $n \geq 1$. So we have

$$x_i, y_i \in \left[M, \frac{M}{M - 1}\right], \qquad i = 1, 2, \dots, m + 1,$$

where $M = \min\{\mu, \nu/(\nu - 1)\} > 1$. Then

$$M = 1 + \frac{M}{M/(M - 1)} \leq x_{m+2} = 1 + \frac{x_1}{y_{m+1}} \leq M + \frac{M/(M - 1)}{M} = \frac{M}{M - 1},$$
$$M = 1 + \frac{M}{M/(M - 1)} \leq y_{m+2} = 1 + \frac{y_1}{x_{m+1}} \leq M + \frac{M/(M - 1)}{M} = \frac{M}{M - 1},$$

By induction, we get

$$x_i, y_i \in \left[M, \frac{M}{M-1}\right], \qquad i = 1, 2, \ldots,$$

This completes the proof of Theorem 3.1. □

## 4. The Case $A > 1$ and $B > 1$

This section concerns itself with the global asymptotic stability of the unique equilibrium point of (1.4) for the case $A > 1$, $B > 1$.

**Theorem 4.1.** *Suppose that $A > 1$, $B > 1$. Then for $n = km + i$, every positive solution $\{(x_n, y_n)\}$ of (1.4) satisfies*

$$A \le x_{km+i} \le \frac{1}{B^k}\left(x_{i-k} - \frac{AB}{B-1}\right) + \frac{AB}{B-1}, \quad i = k-m, k-m+1, \ldots, k, \quad k \in \{1, 2, \ldots, \},$$

$$B \le y_{km+i} \le \frac{1}{A^k}\left(y_{i-k} - \frac{AB}{A-1}\right) + \frac{AB}{A-1}, \quad i = k-m, k-m+1, \ldots, k, \quad k \in \{1, 2, \ldots, \}.$$

*Proof.* Let $\{(x_n, y_n)\}$ be arbitrary positive solution of (1.4). Clearly, we have $x_n \ge A > 1$, $y_n \ge B > 1$ for $n = 1, 2, \ldots$. Moreover using (1.4), we have

$$\begin{aligned} x_{n+1} &= A + \frac{x_{n-m}}{y_n} \le A + \frac{1}{B}x_{n-m}, \\ y_{n+1} &= B + \frac{y_{n-m}}{x_n} \le B + \frac{1}{A}y_{n-m}, \qquad n \ge 1. \end{aligned} \tag{4.1}$$

Let $v_n, w_n$ be the solution of the equation

$$v_{n+1} = A + \frac{1}{B}v_{n-m}, \quad w_{n+1} = B + \frac{1}{A}w_{n-m}, \quad n \ge 1. \tag{4.2}$$

such that

$$\begin{aligned} &v_{-m} = x_{-m}, \quad v_{1-m} = x_{1-m}, \quad \ldots, \quad v_0 = x_0, \\ &w_{-m} = y_{-m}, \quad w_{1-m} = y_{1-m}, \quad \ldots, \quad w_0 = y_0. \end{aligned} \tag{4.3}$$

We prove by induction that for $n = km + i$,

$$\begin{aligned} &x_{km+i} \le v_{km+i}, \qquad y_{km+i} \le w_{km+i}, \qquad k \in \{1, 2, \ldots, \}, \\ &i = k-m, k-m+1, \ldots, k. \end{aligned} \tag{4.4}$$

Suppose that (4.4) is true for $k = p \ge 1$. Then from (4.1) and (4.2) we get

$$\begin{aligned} x_{(p+1)m+i} &\le A + \frac{1}{B}x_{pm+i-1} \le A + \frac{1}{B}v_{pm+i-1} = v_{(p+1)m+i}, \\ y_{(p+1)m+i} &\le B + \frac{1}{A}y_{pm+i-1} \le B + \frac{1}{A}w_{pm+i-1} = w_{(p+1)m+i}. \end{aligned}$$

Therefore (4.4) is true. From (4.2) and (4.3), we have

$$\begin{aligned} &v_{km+i} = \frac{1}{B^k}\left(x_{i-k} - \frac{AB}{B-1}\right) + \frac{AB}{B-1}, \qquad k \in \{1,2,\ldots,\}, \\ &i = k-m, k-m+1, \ldots, k. \end{aligned} \tag{4.5}$$

$$\begin{aligned} &w_{km+i} = \frac{1}{A^k}\left(y_{i-k} - \frac{AB}{A-1}\right) + \frac{AB}{A-1}, \qquad k \in \{1,2,\ldots,\}, \\ &i = k-m, k-m+1, \ldots, k. \end{aligned} \tag{4.6}$$

Then from (4.4), (4.5) and (4.6), the statement of Theorem 4.1 is true. □

**Theorem 4.2.** *Suppose that $A > 1$, $B > 1$. Then the positive equilibrium*

$$(\bar{x}, \bar{y}) = \left(\frac{AB-1}{B-1}, \frac{AB-1}{A-1}\right)$$

*of* (1.4) *is globally asymptotically stable.*

*Proof.* The linearized equation of system (1.4) about the equilibrium point $(\bar{x}, \bar{y}) = \left(\frac{AB-1}{B-1}, \frac{AB-1}{A-1}\right)$ is

$$\Psi_{n+1} = E\Psi_n \tag{4.7}$$

where

$$\Psi_n = \begin{pmatrix} x_n \\ \vdots \\ x_{n-m} \\ y_n \\ \vdots \\ y_{n-m} \end{pmatrix},$$

$$E = (e_{ij})_{(2m+2)\times(2m+2)} = \begin{pmatrix} 0 & \ldots & 0 & \frac{1}{\bar{y}} & -\frac{\bar{x}}{\bar{y}^2} & \ldots & 0 & 0 \\ 1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\ & \ddots & & & & & & \\ 0 & \ldots & 1 & 0 & 0 & \ldots & 0 & 0 \\ -\frac{\bar{y}}{\bar{x}^2} & \ldots & 0 & 0 & 0 & \ldots & 0 & \frac{1}{\bar{x}} \\ 0 & \ldots & 0 & 0 & 1 & \ldots & 0 & 0 \\ & & & & & \ddots & & \\ 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 \end{pmatrix}.$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_{2m+2}$ denote the $2m+2$ eigenvalues of Matrix $E$. Let $D = \operatorname{diag}(d_1, d_2, \ldots, d_{2m+2})$ be a diagonal matrix, where $d_1 = d_{m+2} = 1$, $d_{1+k} = d_{m+2+k} = 1 - k\varepsilon$, $1 \le k \le m$. and

$$\varepsilon = \min\left\{\frac{1}{m}, \frac{1}{m}\left(1 - \frac{\bar{y}}{\bar{y}^2 - \bar{x}}\right), \frac{1}{m}\left(1 - \frac{\bar{x}}{\bar{x}^2 - \bar{y}}\right)\right\}.$$

Clearly, $D$ is invertible. Computing $DED^{-1}$, we obtain

$$
DED^{-1} = \begin{pmatrix}
0 & \cdots & 0 & \frac{d_1}{\bar{y}} d_{m+1}^{-1} & -\frac{\bar{x}}{\bar{y}^2} d_1 d_{m+2}^{-1} & \cdots & 0 & 0 \\
d_2 d_1^{-1} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
 & \ddots & & & & & & \\
0 & \cdots & d_{m+1} d_m^{-1} & 0 & 0 & \cdots & 0 & 0 \\
-\frac{\bar{y}}{\bar{x}^2} d_{m+2} d_1^{-1} & \cdots & 0 & 0 & 0 & \cdots & 0 & \frac{d_{m+2}}{\bar{x}} d_{2m+2}^{-1} \\
0 & \cdots & 0 & 0 & d_{m+3} d_{m+2}^{-1} & \cdots & 0 & 0 \\
 & & & & & \ddots & & \\
0 & \cdots & 0 & 0 & 0 & \cdots & d_{2m+2} d_{2m+1}^{-1} & 0
\end{pmatrix}
$$

The following two chains of inequalities

$$
d_{m+1} > d_m > \ldots > d_2 > 0, \qquad d_{2m+2} > d_{2m+1} > \ldots > d_{m+3} > 0
$$

imply that

$$
\begin{aligned}
& d_2 d_1^{-1} < 1, \ \ d_3 d_2^{-1} < 1, \ \ldots, \ d_{m+1} d_m^{-1} < 1, \\
& d_{m+3} d_{m+2}^{-1} < 1, \ \ d_{m+4} d_{m+3}^{-1} < 1, \ \ldots, \ d_{2m+2} d_{2m+1}^{-1} < 1.
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
\frac{d_1}{\bar{y}} d_{m+1}^{-1} + \frac{\bar{x}}{\bar{y}^2} d_1 d_{m+2}^{-1} &= \frac{1}{\bar{y}} d_{m+1}^{-1} + \frac{\bar{x}}{\bar{y}^2} = \frac{1}{\bar{y}(1-m\varepsilon)} + \frac{\bar{x}}{\bar{y}^2} < 1, \\
\frac{d_{m+2}}{\bar{x}} d_{2m+2}^{-1} + \frac{\bar{y}}{\bar{x}^2} d_{m+2} d_1^{-1} &= \frac{1}{\bar{x}} d_{2m+2}^{-1} + \frac{\bar{y}}{\bar{x}^2} = \frac{1}{\bar{x}(1-m\varepsilon)} + \frac{\bar{y}}{\bar{x}^2} < 1.
\end{aligned}
$$

It is well known that $E$ has the same eigenvalues as $DED^{-1}$, we obtain that

$$
\begin{aligned}
\max_{1 \le k \le 2m+2} |\lambda_k| = \|DED^{-1}\| \\
&= \max\Big\{ d_2 d_1^{-1}, \ldots, d_{m+1} d_m^{-1}, d_{m+3} d_{m+2}^{-1}, \ldots, d_{2m+2} d_{2m+1}^{-1}, \\
&\qquad \frac{d_1}{\bar{y}} d_{m+1}^{-1} + \frac{\bar{x}}{\bar{y}^2} d_1 d_{m+2}^{-1}, \frac{d_{m+2}}{\bar{x}} d_{2m+2}^{-1} + \frac{\bar{y}}{\bar{x}^2} d_{m+2} d_1^{-1} \Big\} \\
&< 1
\end{aligned}
$$

Hence, the equilibrium of (4.1) is locally asymptotically stable. This implies that the equilibrium $(\bar{x}, \bar{y})$ of (1.4) is locally asymptotically stable.

Next we prove that every positive solution $(x_n, y_n)$ of (1.4) converges to $(\bar{x}, \bar{y})$. Let $(x_n, y_n)$ be an arbitrary positive solution of (1.4) Let

$$
\begin{aligned}
L_1 &= \lim_{n\to\infty} \sup\{x_n, x_{n+1}, \ldots\}, \qquad l_1 = \lim_{n\to\infty} \inf\{x_n, x_{n+1}, \ldots\}, \\
L_2 &= \lim_{n\to\infty} \sup\{y_n, y_{n+1}, \ldots\}, \qquad l_2 = \lim_{n\to\infty} \inf\{y_n, y_{n+1}, \ldots\}.
\end{aligned}
$$

From Theorem 4.1, we have $0 < A \le l_1 \le L_1 < +\infty$ and $0 < B \le l_2 \le L_2 < +\infty$. This and (1.4) imply

$$\begin{aligned} L_1 &\le A + \frac{L_1}{l_2}, \qquad & L_2 &\le B + \frac{L_2}{l_1}, \\ l_1 &\ge A + \frac{l_1}{L_2}, \qquad & l_2 &\ge B + \frac{l_2}{L_1}. \end{aligned}$$

Which can be written as

$$\begin{aligned} L_1 l_2 &\le A l_2 + L_1, \qquad & L_2 l_1 &\le B l_1 + L_2, \\ l_1 L_2 &\ge A L_2 + l_1, \qquad & l_2 L_1 &\ge B L_1 + l_2. \end{aligned}$$

From them we have

$$L_1 L_2 \le l_1 l_2.$$

So

$$L_1 L_2 = l_1 l_2. \tag{4.8}$$

We claim that

$$L_1 = l_1, \quad L_2 = l_2. \tag{4.9}$$

Suppose on contrary that $l_1 < L_1$. Then from (4.8) we have $L_1 L_2 = l_1 l_2 < L_1 l_2$ and so $L_2 < l_2$, which is a contradiction. So $L_1 = l_1$. Similarly, we can prove that $L_2 = l_2$. Therefore, (4.9) are true. So $\lim_{n\to\infty} x_n = \bar{x}, \lim_{n\to\infty} y_n = \bar{y}$. Hence the equilibrium $(\bar{x}, \bar{y})$ is globally exponentially stable. □

## 5. Conclusion and remarks

In this paper, we study the system of two nonlinear difference equations (1.4) under different conditions. When $A < 1$ and $B < 1$, we concern ourselves with the asymptotic behavior of positive solution to (1.4). We show that every positive solution is bounded and persistence if $A = B = 1$. Finally we investigate the unique positive equilibrium which is globally asymptotically stable if $A > 1$ and $B > 1$.

At the end we propose the following open problem.

**Open problem.** Let $A > 1$ and $B < 1$ or $A < 1$ and $B > 1$, discuss the behavior of positive solution of system (1.4).

## References


**1.** Amleha A. M., Grovea E. A, Ladasa G. and Georgiou D.A, *On the recursive sequence* $x_{n+1} = \alpha + (x_{n-1}/x_n)$. Journal of Mathematical Analysis and Applications, **233** (1999), 790–798.

**2.** EL-Owaidy H. M., Ahmed A. M. and Mousa M. S., *On asymptotic behavior of the difference equation* $x_{n+1} = \alpha + (x_{n-1}^p/x_n^p)$. J. Appl. Math. Comput. **12** (2003), 31–37.

**3.** Stević S., *On the recursive sequences* $x_{n+1} = \alpha + (x_{n-1}^p/x_n^p)$. J. Appl. Math. Comput. **18** (2005), 229–234.

**4.** Papaschinopoulos G., Kariakouli P. Ch. and Hatzifilippidis V., *Oscillation and asymptotic stability of a nonlinear difference equation.* Nonlinear Analysis **47** (2001), 4885–4896.

**5.** DeVault R., Ladas G. and Schultz S. W., *On the recursive sequence* $x_{n+1} = A/x_n + 1/x_{n-2}$. Proc. Amer. Math. Soc. **126** (1998), 3257–3261.
**6.** ———, *Necessary and sufficient conditions the boundedness of* $x_{n+1} = A/x_n^p + B/x_{n-1}^q$. J. Difference Equations Appl. **3** (1998), 259–266.
**7.** Abu-Saris R. M. and DeVault R., *Global stability of* $y_{n+1} = A + y_n/y_{n-k}$. Appl. Math. Lett. **16** (2003), 173–178.
**8.** Li W. T. and Sun H. R., *Dynamic of a rational difference equation*. Appl. Math. Compt. **163** (2005), 577–591.
**9.** Su Y. H. and Li W. T., *Global attractivity of a higher order nonlinear difference equation*. J. Differ. Equat. Appl. **11** (2005), 947–958.
**10.** Hu L. X. and Li W. T., *Global stability of a rational difference equation*, Appl. Math. Compt. **190** (2007), 1322–1327.
**11.** Clark D., Kulenovic M. R. S. and Selgrade J. F. *Global asymptotic behavior of a two-dimensional difference equation modelling competition*. Nonlinear Analysis **52** (2003), 1765–1776.
**12.** Clark D. and Kulenovic M. R. S., *A coupled system of rational difference equations*. Comput. .Math. Appl. **43** (2002), 849–867.
**13.** Papaschinopoulos G. and Papadopoulos B. K., *On the fuzzy difference equation* $x_{n=1} = A + x_n/x_{n-m}$. Fuzzy Sets and Systems **129** (2002), 73–81.
**14.** Camouzis E. and Papaschinopoulos G., *Global asymptotic behavior of positive solutions on the system of rational difference equations* $x_{n+1} = 1 + x_n/y_{n-m}$, $y_{n+1} = 1 + y_n/x_{n-m}$. Applied Mathematics Letters **17** (2004), 733–737.
**15.** Papaschinopoulos G. and Schinas C. J., *On a system of two nonlinear difference equations*. J. Math. Anal. Appl. **219** (1998), 415–426.
**16.** ———, *On the system of two nonlinear difference equations* $x_{n+1} = A + x_{n-1}/y_n$, $y_{n+1} = A + y_{n-1}/x_n$,. International Journal of Mathematics & Mathematical Sciences **12** (2000), 839–848.
**17.** Yang X. F., *On the system of rational difference equations* $x_n = A + y_{n-1}/x_{n-p}y_{n-q}$, $y_n = A + x_{n-1}/x_{n-r}y_{n-s}$. Journal of Mathematical Analysis and Applications **307** (2005), 305–311.
**18.** Zhang Y., Yang X. F., Evans D. J. and Zhu C., *On the nonlinear difference equation system* $x_{n+1} = A + y_{n-m}/x_n$, $y_{n+1} = A + x_{n-m}/y_n$. Computers and Mathematics with Applications **53** (2007), 1561–1566.

Qianhong Zhang, Guizhou Key Laboratory of Economics System Simulation, Guizhou University of Finance and Economics, Guiyang, Guizhou 550004, P. R. China, *e-mail*: `zqianhong68@163.com`

Lihui Yang, Department of Mathematics, Hunan City University, Yiyang, Hunan 413000, P. R. China, *e-mail*: `ll.hh.yang@gmail.com`

Jingzhong Liu, Computer and Information Science Department, Hunan Institute of Technology, Hengyang, Hunan 421002, P. R. China, *e-mail*: `hnhyls@126.com`