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ON THE CONVERGENCE ALMOST EVERYWHERE OF DOUBLE SERIES WITH RESPECT TO DIAGONAL BLOCK-ORTHONORMAL SYSTEMS

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Abstract. The diagonal double block-orthonormal system is introduced. The two-dimensional generalization of Menshov-Rademacher's and V.F. Gaposhkin's theorems on the almost everywhere convergence of series with respect to block-orthonormal systems is proved.

Block-orthonormal systems were introduced by Gaposhkin [2]. He proved, that the Menshov-Rademacher's theorem [1] and the strong law of large numbers are valid for such systems in certain conditions. In [3] were obtained some results on convergence and summability of series with respect to block-orthonormal systems. In particular, Menshov-Rademacher's and Gaposhkin's theorems were generalized and the exact Weyl multipliers for the convergence and summability almost everywhere of series with respect to block-orthogonal systems were established in the cases, when Menshov-Rademacher's and Gaposhkin's theorems are not true.

The two-dimensional analog of Menshov-Rademacher's theorem was obtained in [4]. In [5] was considered the almost everywhere convergence of multiple orthogonal series.

In the present paper it will be introduced a diagonal block-orthonormal systems and it will be considered the almost everywhere convergence of double series with respect to diagonal block-orthonormal systems.

Definition 1. Let $\{M_k\}$ and$\{N_k\}$ be the increasing sequences of natural numbers and $\Delta_k = ([1, M_{k+1}] \times [1, N_{k+1}])\backslash([1, M_k] \times [1, N_k])$, $(k \geq 1)$. Let $\{\varphi_{mn}\}$ be a system of functions from $L^2((0,1)^2)$. The system $\{\varphi_{mn}\}$ will be called a *diagonal Δ_k-orthonormal system (Δ_k-ONS)* if:

1. $\|\varphi_{mn}\|_2 = 1$, $m = 1, 2, \ldots, n = 1, 2, \ldots$;
2. $(\varphi_{ij}, \varphi_{pq}) = 0$, for $(i,j), (p,q) \in \Delta_k$, $(i,j) \neq (p,q)$, $(k \geq 1)$.

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Let the sequences $\{M_k\}$, $\{N_k\}$ be fixed and $\{\varphi_{mn}\}$ be a diagonal Δ_k-ONS. Let the double series

$$\sum_{m,n=1}^{\infty} a_{mn}\varphi_{mn}(x,y) \tag{1}$$

Is given, where $\sum_{m,n=1}^{\infty} a_{mn}^2 < \infty$.

Under the convergence of the series (1) it is understood the convergence in Pringscame's sense, that is the existence of the limit

$$\lim_{M,N\to\infty} \sum_{m=1}^{M}\sum_{n=1}^{N} a_{mn}\varphi_{mn}(x,y), \tag{2}$$

as M and N independently approaches infinity.

Definition 2. Let $\{\omega\,(m,n)\}$ be a sequence of positive numbers, for which $\omega\,(m,n) \le \omega\,(m,n+1)$ and $\omega\,(m,n) \le \omega\,(m+1,n)$ $(m,n = 1,2,\ldots)$. The sequence $\{\omega\,(m,n)\}$ will be called the *Weyl multiplier* for the convergence almost everywhere (a. e.) of series (1) with respect to diagonal Δ_k-ONS $\{\varphi_{mn}\}$ if the convergence of the series $\sum_{m,n=1}^{\infty} a_{mn}^2\omega(m,n) < \infty$ guarantees the existence of the limit (2) a. e. on $(0,1)^2$.

In this paper, the logarithms are to the base 2.

Theorem. *Let the sequences $\{M_k\}$, $\{N_k\}$ be fixed and $\{\omega_1(m)\}$, $\{\omega_2(n)\}$ be the nondecreasing sequences of positive numbers. In order that a double sequence $\{\omega_1(m)\omega_2(n)\}$ be the Weyl multiplier for the convergence a. e. of series* (1) *with respect to all diagonal Δ_k-ONS $\{\varphi_{mn}\}$, it is necessary and sufficient that the following two conditions be fulfilled:*

$$\sum_{p,q=1}^{\infty} \frac{1}{\omega_1(M_p)\omega_2(N_q)} < \infty, \tag{3}$$

$$\log^2 m = O(\omega_i(m)),\ \ i = 1,2,\ \ (m\to\infty). \tag{4}$$

Below we shall use the following lemma, which is the two-dimensional analog of well-known lemma: (see [1, Lemma 2.3.1], [4, lemma 1]).

Lemma 1. *Let $\{\varphi_{mn}\}$ be a orthonormal system from $L^2((0,1)^2)$. Then for all numbers $\{a_{mn}\}_{0\le m\le M, 0\le n\le N}$ are fulfilled:*

$$\int_0^1\int_0^1 \max_{\substack{0\le m\le M\\ 0\le n\le N}} \left|\sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y \le c\log^2(M+2)\log^2(N+2)\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2 \tag{5}$$

$$\int_0^1\int_0^1 \max_{0\le m\le M}\left|\sum_{i=0}^{m}\sum_{j=0}^{N} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y \le c\log^2(M+2)\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2, \tag{6}$$

$$\int_0^1\int_0^1 \max_{0\le n\le N}\left|\sum_{i=0}^{M}\sum_{j=0}^{n} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y \le c\log^2(N+2)\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2. \tag{7}$$

For (5), (6) and (7) we have generalizations of Kantorovich ([1, p. 89]):

$$\begin{aligned}\int_0^1\int_0^1 \max_{\substack{0\le m\le M,\\ 0\le n\le N}}\left|\sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y&\\ \le c\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2\log^2(i+2)\log^2(j+2)&,\end{aligned} \tag{8}$$

$$\int_0^1\int_0^1 \max_{0\le m\le M}\left|\sum_{i=0}^{m}\sum_{j=0}^{N} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y \le c\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2\log^2(i+2), \tag{9}$$

$$\int_0^1\int_0^1 \max_{0\le n\le N}\left|\sum_{i=0}^{M}\sum_{j=0}^{n} a_{ij}\varphi_{ij}(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y \le c\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}^2\log^2(i+2). \tag{10}$$

Proof of Theorem. Sufficiency. Let for sequence $\{\omega_1(m)\omega_2(n)\}$ the conditions (3), (4) are fulfilled and let for sequence $\{a_{mn}\}$ have:

$$\sum_{m,n=1}^{\infty} a_{mn}^2\omega_1(m)\omega_2(n) < \infty.$$

Let $\{\varphi_{mn}\}$ be arbitrary diagonal Δ_k-ONS. In first we shall prove that the limit

$$\lim_{p,q\to\infty} S_{M_p,N_q}(x,y) = \sum_{m=1}^{M_p}\sum_{n=1}^{N_q} a_{mn}\varphi_{mn}(x,y) \tag{11}$$

exists almost everywhere on $(0,1)^2$.

Without loss of generality it can be assumed that $M_0 = N_0 = 0$ and $\omega_1(0) = \omega_2(0) = 1$. Then

$$\begin{aligned}&\left|S_{M_{p+s},N_{q+r}}(x,y) - S_{M_p,N_q}(x,y)\right|\\ &\quad= \left|\sum_{m=M_p+1}^{M_{p+s}}\sum_{n=1}^{N_{q+r}} a_{mn}\varphi_{mn}(x,y) + \sum_{m=1}^{M_p}\sum_{n=N_q+1}^{N_{q+r}} a_{mn}\varphi_{mn}(x,y)\right|\end{aligned}$$

$$\leq \sum_{i=p}^{\infty}\sum_{j=0}^{\infty}\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y)\right| + \sum_{i=0}^{\infty}\sum_{j=q}^{\infty}\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y)\right| = \mathrm{I}_p(x,y)+\mathrm{J}_q(x,y).$$

We shall prove that the double series

$$\sum_{i,j=0}^{\infty}\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y)\right| \tag{12}$$

converges a. e. on $(0,1)^2$. Indeed, we have

$$\sum_{i,j=0}^{\infty}\int_0^1\int_0^1\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y)\right|\mathrm{d}x\,\mathrm{d}y$$

$$\leq \sum_{i,j=0}^{\infty}\left(\int_0^1\int_0^1\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y)\right|^2\mathrm{d}x\,\mathrm{d}y\right)^{\frac12}$$

$$= \sum_{i,j=0}^{\infty}\left(\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}} a_{mn}^2\right)^{\frac12} \leq c\left(\sum_{i,j=1}^{\infty} a_{mn}^2\omega_1(m)\omega_2(n)\right)^{\frac12}<\infty$$

Hence Levi's theorem implies, that series (12) converges a. e. on $(0,1)^2$. Then almost everywhere on $(0,1)^2$ we have

$$\lim_{p\to\infty}\mathrm{I}_p(x,y)=0 \text{ and } \lim_{q\to\infty}J_q(x,y)=0.$$

Therefore the limit (11) exists almost everywhere on $(0,1)^2$.

Let k, l be the natural numbers, for which

$$M_p<k\leq M_{p+1},\quad N_q<l\leq N_{q+1}.$$

We have

$$\max_{\substack{M_p<k\leq M_{p+1}\\ N_q<l\leq N_{q+1}}}\left|S_{k,l}(x,y)-S_{M_p,N_q}(x,y)\right|$$

$$\leq \max_{N_q<l\leq N_{q+1}}\left|\sum_{m=1}^{M_p}\sum_{n=N_q+1}^{l} a_{mn}\varphi_{mn}(x,y)\right| + \max_{M_p<k\leq M_{p+1}}\left|\sum_{m=M_p+1}^{k}\sum_{n=1}^{N_q} a_{mn}\varphi_{mn}(x,y)\right|$$

$$+\max_{\substack{M_p<k\leq M_{p+1}\\ N_q<l\leq N_{q+1}}}\left|\sum_{m=M_p+1}^{k}\sum_{n=N_q+1}^{l} a_{mn}\varphi_{mn}(x,y)\right|$$

$$\le \sum_{i=0}^{\infty} \sup_{N_q\le l_1<l_2<\infty} \left| \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=l_1+1}^{l_2} a_{mn}\varphi_{mn}(x,y) \right|$$
$$+\sum_{j=0}^{\infty} \sup_{M_p\le k_1<k_2<\infty} \left| \sum_{m=k_1+1}^{k_2} \sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y) \right|$$
$$+ \max_{\substack{M_p<k\le M_{p+1}\\ N_q<l\le N_{q+1}}} \left| \sum_{m=M_p+1}^{k} \sum_{n=N_q+1}^{l} a_{mn}\varphi_{mn}(x,y) \right|$$
$$= \sum_{i=0}^{\infty} \alpha_i^q(x,y) + \sum_{j=0}^{\infty} \beta_j^p(x,y) + \delta_{p,q}(x,y).$$

It's clear, that the sequences

$$\alpha_q(x,y) = \sum_{i=0}^{\infty} \alpha_i^q(x,y) \text{ and } \beta_p(x,y) = \sum_{j=0}^{\infty} \beta_j^p(x,y)$$

are increasing sequences. Show that a. e. on $(0,1)^2$

$$\lim_{q\to\infty} \alpha_q(x,y) = 0 \text{ and } \lim_{p\to\infty} \beta_p(x,y) = 0.$$

Indeed, using lemma we have:

$$\sum_{i=0}^{\infty} \int_0^1 \int_0^1 \alpha_i^q(x,y)\mathrm{d}x\,\mathrm{d}y \le \sum_{i=0}^{\infty} \left(\int_0^1 \int_0^1 \left[\alpha_i^q(x,y)\right]^2 \mathrm{d}x\,\mathrm{d}y \right)^{\frac{1}{2}}$$
$$\le \sum_{i=0}^{\infty} (\int_0^1 \int_0^1 \sup_{N_q\le l_1<l_2<\infty} | \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=N_q+1}^{l_2} a_{mn}\varphi_{mn}(x,y)$$
$$- \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=N_q+1}^{l_1} a_{mn}\varphi_{mn}(x,y)|^2 \mathrm{d}x\,\mathrm{d}y)^{\frac{1}{2}}$$
$$\le c\sum_{i=0}^{\infty} \left(\int_0^1 \int_0^1 \sup_{N_q<l} \left| \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=N_q+1}^{l} a_{mn}\varphi_{mn}(x,y) \right|^2 \mathrm{d}x\,\mathrm{d}y \right)^{\frac{1}{2}}$$
$$\le c\sum_{i=0}^{\infty} \left[\int_0^1 \int_0^1 \left(\sum_{j=q}^{\infty} \left| \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=N_j+1}^{N_{j+1}} a_{mn}\varphi_{mn}(x,y) \right| \right)^2 \mathrm{d}x\,\mathrm{d}y \right]^{\frac{1}{2}}$$
$$+ c\sum_{i=0}^{\infty} \left[\int_0^1 \int_0^1 \sum_{j=q}^{\infty} \max_{N_j<l\le N_{j+1}} \left| \sum_{m=M_i+1}^{M_{i+1}} \sum_{n=N_j+1}^{l} a_{mn}\varphi_{mn}(x,y) \right|^2 \mathrm{d}x\,\mathrm{d}y \right]^{\frac{1}{2}}$$

$$\leq c\sum_{i=0}^{\infty}\left[\left(\sum_{j=q}^{\infty}\int_0^1\int_0^1\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}}a_{mn}\varphi_{mn}(x,y)\right|^2\mathrm{d}x\,\mathrm{d}y\right)^{\frac12}\right.$$

$$\left.+\left(\sum_{j=q}^{\infty}\int_0^1\int_0^1\max_{N_j<l\leq N_{j+1}}\left|\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{l}a_{mn}\varphi_{mn}(x,y)\right|^2\mathrm{d}x\,\mathrm{d}y\right)^{\frac12}\right]$$

$$\leq c\sum_{i=0}^{\infty}\left[\sum_{i=q}^{\infty}\left(\sum_{m=M_i^{+1}}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}}\alpha_{mn}^2\right)^{\frac12}+\left(\sum_{i=q}^{\infty}\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}}\alpha_{mn}^2\log^2(n+2)\right)^{\frac12}\right]$$

$$\leq c\left(\sum_{i=0}^{\infty}\sum_{i=q}^{\infty}\left(\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}}\alpha_{mn}^2\right)\omega_1(M_i)\omega_2(N_j)\right)^{\frac12}\cdot\left(\sum_{i=0}^{\infty}\sum_{i=q}^{\infty}\frac{1}{\omega_1(M_i)\omega_2(N_j)}\right)^{\frac12}$$

$$\leq c\left(\sum_{i=0}^{\infty}\sum_{i=q}^{\infty}\sum_{m=M_i+1}^{M_{i+1}}\sum_{n=N_j+1}^{N_{j+1}}\alpha_{mn}^2\log^2(n+2)\omega_1(M_i)\right)^{\frac12}\cdot\left(\sum_{i=0}^{\infty}\frac{1}{\omega_1(M_i)}\right)^{\frac12}$$

$$\leq c\left(\sum_{m=1}^{\infty}\sum_{n=N_q+1}^{\infty}\alpha_{mn}^2\omega_1(m)\omega_2(n)\right)^{\frac12},$$

hence

$$\lim_{q\to\infty}\int_0^1\int_0^1\alpha_q(x,y)\mathrm{d}x\,\mathrm{d}y=0.$$

Then by Fatou's theorem

$$\lim_{q\to\infty}\alpha_q(x,y)=0\ \text{a. e. on}\ (0,1)^2. \tag{13}$$

Similarly we obtain

$$\lim_{p\to\infty}\beta_p(x,y)=0\ \text{a. e. on}\ (0,1)^2. \tag{14}$$

Now we prove that

$$\lim_{p,q\to\infty}\delta_{p,q}(x,y)=0\ \text{a. e. on}\ (0,1)^2. \tag{15}$$

Indeed, using inequality (8) we get

$$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\int_0^1\int_0^1\delta_{p,q}^2(x,y)\mathrm{d}x\,\mathrm{d}y$$

$$\leq c\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{m=M_p+1}^{M_{p+1}}\sum_{n=N_q+1}^{N_{q+1}}a_{mn}^2\log^2(m+2)\log^2(n+2)$$

$$\le c \sum_{m,n=1}^{\infty} a_{mn}^2 \omega_1(m)\omega_2(n) < \infty,$$

hence $\sum_{p,q=0}^{\infty} \delta_{p,q}^2(x,y) < \infty$ almost everywhere on $(0,1)^2$. Then we obtain (15). Therefore taking into account (13), (14) and (15) we get

$$\max_{M_p<k\le M_{p+1},\ N_q<l\le N_{q+1}} \left|S_{k,l}(x,y) - S_{M_p,N_q}(x,y)\right| = 0$$

almost everywhere on $(0,1)^2$. Finally taking into account (11) we finished proof of sufficiency.

Necessity. a) Let

$$\sum_{p,q=1}^{\infty} \frac{1}{\omega_1(M_p)\omega_2(N_q)} = \infty\,.$$

Without loss of generality it can be assumed that

$$\sum_{p=1}^{\infty} \frac{1}{\omega_1(M_p)} = \infty\,.$$

Then there exist numbers $c_p > 0$ such that

$$\sum_{p,q=1}^{\infty} c_p^2\,\omega_1(M_p) < \infty \text{ and } \sum_{p=1}^{\infty} c_p = \infty.$$

Take $a_{M_p,N_1} = c_p$, $(p = 1,2,\ldots)$, $a_{mn} = 0$, $((m,n) \ne (M_p,N_1),\ m \in \mathbb{N},\ n \in \mathbb{N},\ p \in \mathbb{N})$. Let $\varphi_{M_pN_1}(x,y) = 1$, $(p = 1,2,\ldots)$, $(x,y) \in (0,1)^2$ and choose as other functions an arbitrary ONS orthogonal to 1. The system $\{\varphi_{mn}\}$ is diagonal Δ_k-ONS , for which

$$\sum_{m,n=1}^{\infty} a_{mn}\varphi_{mn}(x,y) = \sum_{p=1}^{\infty} c_p = \infty \ (x,y) \in (0,1)^2$$

Though

$$\sum_{m,n=1}^{\infty} a_{mn}^2\omega_1(m)\omega_2(n) = \sum_{p=1}^{\infty} c_p^2\,\omega_1(M_p)\omega_2(N_1) < \infty.$$

b) Let condition (4) is not fulfilled. Without loss of generality it can be assumed that the condition $\log^2 m = O(\omega_1(m))$, $(m \to \infty)$ is not fulfilled. Then there exist (see [3, Theorem 1.]), numbers b_m and $(M_p, M_{p+1}]$-ONS $\{\varphi_m\}$ such that

$$\sum_{m=1}^{\infty} b_m^2\omega_1(m) < \infty, \tag{16}$$

though

$$\sum_{m=1}^{\infty} b_m\varphi_m(x) \tag{17}$$

diverges a. e. on $(0,1)$.

Take $a_{m,1}=b_m$, $(m=1,2,\ldots)$, $a_{mn}=0$, $(m\in\mathbb{N}, n\ge 2)$. Let $\{\psi_n\}$ be an ONS from $L^2(0,1)$ such that $\psi_1(y)=1$, $y\in(0,1)$. The system $\varphi_{mn}(x,y)=\varphi_m(x)\psi_n(y)$ is a diagonal Δ_k-orthonormal system. Then taking into account (16), (17) we have

$$\sum_{m,n=1}^{\infty} a_{mn}^2\omega_1(m)\omega_2(n)=\omega_2(1)\sum_{m=1}^{\infty} b_m^2\omega_1(m)<\infty\,,$$

though the series

$$\sum_{m,n=1}^{\infty} a_{mn}\varphi_{mn}(x,y)=\sum_{m=1}^{\infty} b_m\varphi_m(x)\psi_1(y)$$

diverges a. e. on $(0,1)^2$. □

Corollary. *If we take $\omega_1(m)=\omega_2(m)=\log^2 m$ then we obtain the following theorem:*

a) If

$$\sum_{p,q=1}^{\infty}\frac{1}{\log^2(M_p)\log^2(N_q)}<\infty\,, \tag{18}$$

then for every diagonal Δ_k-ONS $\{\varphi_{mn}\}$ the condition

$$\sum_{m,n=1}^{\infty} a_{mn}^2\log^2 m\log^2 n<\infty$$

guarantees the convergence a. e. on $(0,1)^2$ of the series (1).

b) If however

$$\sum_{p,q=1}^{\infty}\frac{1}{\log^2 M_p\log^2 N_q}=\infty\,,$$

then there exist numbers b_{mn} and diagonal Δ_k-ONS $\{\psi_{mn}\}$ such that the series

$$\sum_{m,n=1}^{\infty} b_{mn}\psi_{mn}(x,y)$$

diverges a. e. on $(0,1)^2$ though

$$\sum_{m,n=1}^{\infty} b_{mn}^2\log^2 m\log^2 n<\infty\,.$$

Remark 1. For example if we take $M_p=\left[2^{p^\alpha}\right]$, $N_q=\left[2^{q^\alpha}\right]$, $\alpha>\frac12$, then the condition (18) is fulfilled. Therefore the two-dimensional analog of Menshov-Rademacher's Theorem (see [4], theorem 1) is fulfilled for any Δ_k-ONS $\{\varphi_{mn}\}$.

If $M_p=\left[2^{p^\alpha}\right]$, $N_q=\left[2^{q^\alpha}\right]$, $0<\alpha\le\frac12$, then $\left\{\log^2 m\log^2 n\right\}$ will be the Weyl multiplier for the convergence a. e. not for each Δ_k-ONS. From proved

Theorem it follows that in that case $\left\{\log^{\frac{1}{\alpha}+\varepsilon} m \log^{\frac{1}{\alpha}+\varepsilon} n\right\}$ $(\varepsilon > 0)$ is the Weyl multiplier.

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