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ON π-IMAGES OF METRIC SPACES

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ABSTRACT. In this paper, we prove that sequence-covering, π-images of metric spaces and spaces with a σ-strong network consisting of fcs-covers are equivalent. We also investigate π-images of separable metric spaces.

1. INTRODUCTION

A study of images of metric spaces is an important question in general topology ([2, 7, 9, 10, 16]). In recent years, π-images of metric spaces cause attention once again ([4, 13, 18, 19]). It is known that a space is a strong-sequence-covering (resp. sequentially-quotient), π-image of a metric space if and only if it has a σ-strong network consisting of cs-covers (resp. cs^*-covers) (see [13], for example). Note that strong-sequence-covering mapping $\Longrightarrow$ sequence-covering mapping $\Longrightarrow$ (if the domain is metric) sequentially-quotient mapping and that cs-cover $\Longrightarrow$ fcs-cover $\Longrightarrow$ cs^*-cover. It is natural to raised the following question.

Question 1.1. Can sequence-covering, π-images of metric spaces be characterized as spaces with a σ-strong network consisting of fcs-covers?

On the other hand, whether sequentially-quotient, π-images of metric spaces and sequence-covering, π-images of metric spaces are equivalent? This question is still open (see [13, Question 3.1.14] or [19, Question 4.4(2)], for example). This leads us to consider the following question.

Question 1.2. Are sequentially-quotient, π-images of separable metric spaces and sequence-covering, π-images of separable metric spaces equivalent?

In this paper, we give a positive answer for Question 1.1. We also investigate π-images of separable metric spaces, and answer Question 1.2 affirmatively.

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Throughout this paper, all spaces are assumed to be Hausdorff, and all mappings are continuous and onto. $\mathbb{N}$ denotes the set of all natural numbers, $\{x_n\}$ denotes a sequence, where the n-th term is x_n. Let X be a space and let A be a subset of X. We say that a sequence $\{x_n\}$ converging to x in X is eventually in A if $\{x_n : n > k\} \bigcup \{x\} \subset A$ for some $k \in \mathbb{N}$. Let $\mathcal{P}$ be a family of subsets of X and let $x \in X$. $\bigcup \mathcal{P}$, $st(x, \mathcal{P})$ and $(\mathcal{P})_x$ denote the union $\bigcup\{P : P \in \mathcal{P}\}$, the union $\bigcup\{P \in \mathcal{P} : x \in P\}$ and the subfamily $\{P \in \mathcal{P} : x \in P\}$ of $\mathcal{P}$ respectively. For a sequence $\{\mathcal{P}_n : n \in \mathbb{N}\}$ of covers of a space X, we abbreviate $\{\mathcal{P}_n : n \in \mathbb{N}\}$ to $\{\mathcal{P}_n\}$. A point $b = (\beta_n)_{n \in \mathbb{N}}$ of a Tychonoff-product space is abbreviated to (β_n), where β_n is the n-th coordinate of b. If $f \colon X \longrightarrow Y$ is a mapping, then $f(\mathcal{P})$ denotes $\{f(P) : P \in \mathcal{P}\}$.

2. π-Images of Metric Spaces

Definition 2.1. Let $f \colon X \longrightarrow Y$ be a mapping.

(1) f is called a strong-sequence-covering mapping ([11]) if for every convergent sequence S in Y, there exists a convergent sequence L in X such that $f(L) = S$.

(2) f is called a sequence-covering mapping ([6]) if for every sequence S converging to y in Y, there exists a compact subset K of X such that $f(K) = S \bigcup \{y\}$.

(3) f is called a sequentially-quotient mapping ([1]) if for every convergent sequence S in Y, there exists a convergent sequence L in X such that $f(L)$ is a subsequence of S.

(4) f is called a compact-covering mapping([15]) if for every compact subset C of Y, there exists a compact subset K of X such that $f(K) = C$.

(5) f is called a π-mapping ([16]), if for every $y \in Y$ and for every neighborhood U of y in Y, $d(f^{-1}(y), X - f^{-1}(U)) > 0$, where X is a metric space with a metric d.

Definition 2.2. Let $\mathcal{P}$ be a cover of a space X.

(1) $\mathcal{P}$ is called an fcs-cover of X ([5]) if for every sequence S converging to x in X, there exists a finite subfamily $\mathcal{P}'$ of $(\mathcal{P})_x$ such that S is eventually in $\bigcup \mathcal{P}'$.

(2) $\mathcal{P}$ is called a cs^*-cover ([13]) if for every convergent sequence S in X, there exist $P \in \mathcal{P}$ and a subsequence S' of S such that S' is eventually in P.

Definition 2.3. (1) Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a cover of a space X, where $\mathcal{P}_x \subset (\mathcal{P})_x$. $\mathcal{P}$ is called a network of X ([15]), if for every $x \in U$ with U open in X, there exists $P \in \mathcal{P}_x$ such that $x \in P \subset U$, where $\mathcal{P}_x$ is called a network at x in X.

(2) Let $\{\mathcal{P}_n\}$ be a sequence of covers of a space X and every $\mathcal{P}_{n+1}$ is an refinement of $\mathcal{P}_n$. $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is called a σ-strong network ([8]), if $\{st(x, \mathcal{P}_n)\}$ is a network at x in X for every $x \in X$.

(3) A σ-strong network $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is called a σ-strong network consisting of (countable) fcs-covers (resp. cs^*-covers) if $\mathcal{P}_n$ is a (countable) fcs-cover (resp. cs^*-cover) for every $n \in \mathbb{N}$.

(4) A σ-strong network $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is called a σ-point-countable strong network if $\mathcal{P}_n$ is point-countable for every $n \in \mathbb{N}$.

Theorem 2.4. *For a space X, the following are equivalent.*
(1) X is a sequence-covering, π-image of a metric space.
(2) X has a σ-strong network consisting of fcs-covers.

Proof. (1)$\Longrightarrow$(2): Let M be a metric space with a metric d, and let $f\colon M \longrightarrow X$ be a sequence-covering, π-mapping. We write $B(a,\varepsilon) = \{b \in M : d(a,b) < \varepsilon\}$ for every $a \in M$, where $\varepsilon > 0$. For every $n \in \mathbb{N}$, put $\mathcal{B}_n = \{B(a,1/n) : a \in M\}$, and put $\mathcal{P}_n = f(\mathcal{B}_n)$, then $\mathcal{P}_n$ is a cover of X.

Claim 1. $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a σ-strong network of X.

It is clear that $\mathcal{P}_{n+1}$ is a refinement of $\mathcal{P}_n$ for every $n \in \mathbb{N}$. We only need to prove that $\{st(x,\mathcal{P}_n)\}$ is a network at x in X for every $x \in X$. Let $x \in U$ with U open in X. Since f is a π-mapping, there exists $n \in \mathbb{N}$ such that $d(f^{-1}(x), M - f^{-1}(U)) > 1/n$. Pick $m \in \mathbb{N}$ such that $m > 2n$. It suffices to prove that $st(x,\mathcal{P}_m) \subset U$. Let $a \in M$ and let $x \in f(B(a,1/m)) \in \mathcal{P}_m$. We claim that $B(a,1/m) \subset f^{-1}(U)$. In fact, if $B(a,1/m) \not\subset f^{-1}(U)$, then pick $b \in B(a,1/m) - f^{-1}(U)$. Note that $f^{-1}(x) \bigcap B(a,1/m) \neq \emptyset$, pick $c \in f^{-1}(x) \bigcap B(a,1/m) \neq \emptyset$, then $d(f^{-1}(x), M - f^{-1}(U)) \le d(c,b) \le d(c,a) + d(a,b) < 2/m < 1/n$. This is a contradiction. So $B(a,1/m) \subset f^{-1}(U)$, thus $f(B(a,1/m)) \subset ff^{-1}(U) = U$. This proves that $st(x,\mathcal{P}_m) \subset U$.

Claim 2. $\mathcal{P}_n$ is an fcs-cover of X for every $n \in \mathbb{N}$.

Let $n \in \mathbb{N}$. Suppose S is a sequence converging to x in X. Since f is sequence-covering, there exists a compact subset K in M such that $f(K) = S \bigcup\{x\}$. Note that $f^{-1}(x) \bigcap K$ is compact in M. There exists a finite subset M' of M such that $f^{-1}(x) \bigcap K \subset \bigcup_{a \in M'} B(a,1/n)$. We can assume that $f^{-1}(x) \bigcap B(a,1/n) \neq \emptyset$ for every $a \in M'$. Put $\mathcal{B} = \{B(a,1/n) : a \in M'\}$ and $B = \bigcup \mathcal{B}$, then $K - B$ is compact in M. Put $\mathcal{P}' = \{f(B(a,1/n)) : a \in M'\}$. Then $\mathcal{P}'$ is a finite subfamily of $(\mathcal{P}_n)_x$. We prove that S is eventually in $\bigcup \mathcal{P}'$ as follows. If not, there exists a subsequence $\{x_k\}$ of S converging to x such that $x_k \notin \bigcup \mathcal{P}'$ for every $k \in \mathbb{N}$. Thus there exists $a_k \in K - B$ such that $f(a_k) = x_k$ for every $k \in \mathbb{N}$. Since $K - B$ is compact in M, there exists a subsequence $\{a_{k_i}\}$ of $\{a_k\}$ such that the sequence $\{a_{k_i}\}$ converges to a point $a \in K - B$. Thus $f(a) \neq x$. This contradicts the continuity of f. So S is eventually in $\bigcup \mathcal{P}'$. This proves that $\mathcal{P}_n$ is an fcs-cover of X.

By the above, X has a σ-strong network $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ consisting of fcs-covers.

(2)$\Longrightarrow$(1): Let X have a σ-strong network $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ consisting of fcs-covers. For every $n \in \mathbb{N}$, put $\mathcal{P}_n = \{P_\alpha : \alpha \in \Lambda_n\}$, and Λ_n is endowed with discrete topology. Put

$$M = \{a = (\alpha_n) \in \Pi_{n\in\mathbb{N}}\Lambda_n : \{P_{\alpha_n}\}\ is\ a\ network\ at\ some\ x_a\ inX\}.$$

Then M, which is a subspace of the product space $\Pi_{n\in\mathbb{N}}\Lambda_n$, is a metric space with metric d described as follows.

Let $a = (\alpha_n), b = (\beta_n) \in M$. If $a = b$, then $d(a, b) = 0$. If $a \neq b$, then $d(a, b) = 1/min\{n \in \mathbb{N} : \alpha_n \neq \beta_n\}$.

Define $f: M \longrightarrow X$ by choosing $f(a) = x_a$ for every $a = (\alpha_n) \in M$, where $\{P_{\alpha_n}\}$ is a network at x_a in X. It is not difficult to check that f is continuous and onto.

Claim 1. f is a π-mapping.

Let $x \in U$ with U open in X. Since $\{\mathcal{P}_n\}$ is a σ-strong network of X, there exists $n \in \mathbb{N}$ such that $st(x, \mathcal{P}_n) \subset U$. Then $d(f^{-1}(x), M - f^{-1}(U)) \geq 1/2n > 0$. In fact, if $a = (\alpha_n) \in M$ such that $d(f^{-1}(x), a) < 1/2n$, then there is $b = (\beta_n) \in f^{-1}(x)$ such that $d(a, b) < 1/n$, so $\alpha_k = \beta_k$ if $k \leq n$. Notice that $x \in P_{\beta_n} \in \mathcal{P}_n$, $P_{\alpha_n} = P_{\beta_n}$, so $f(a) \in P_{\alpha_n} = P_{\beta_n} \subset st(x, \mathcal{P}_n) \subset U$, hence $a \in f^{-1}(U)$. Thus $d(f^{-1}(x), a) \geq 1/2n$ if $a \in M - f^{-1}(U)$, so $d(f^{-1}(x), M - f^{-1}(U)) \geq 1/2n > 0$. This proves that f is a π-mapping.

Claim 2. f is a sequence-covering mapping.

Let $S = \{x_n\}$ be a sequence converging to x in X. For every $n \in \mathbb{N}$, since $\mathcal{P}_n$ is an fcs-cover, there exists a finite subfamily $\mathcal{F}_n$ of $(\mathcal{P}_n)_x$ such that S is eventually in $\bigcup \mathcal{F}_n$. Note that $S - \bigcup \mathcal{F}_n$ is finite. There exists a finite subfamily $\mathcal{G}_n$ of $\mathcal{P}_n$ such that $S - \bigcup \mathcal{F}_n \subset \bigcup \mathcal{G}_n$. Put $\mathcal{F}_n \bigcup \mathcal{G}_n = \{P_{\alpha_n} : \alpha_n \in \Gamma_n\}$, where Γ_n is a finite subset of Λ_n. For every $\alpha_n \in \Gamma_n$, if $P_{\alpha_n} \in \mathcal{F}_n$, put $S_{\alpha_n} = (S \bigcup \{x\}) \bigcap P_{\alpha_n}$, otherwise, put $S_{\alpha_n} = (S - \bigcup \mathcal{F}_n) \bigcap P_{\alpha_n}$. It is easy to see that $S \bigcup \{x\} = \bigcup_{\alpha_n \in \Gamma_n} S_{\alpha_n}$ and $\{S_{\alpha_n} : \alpha_n \in \Gamma_n\}$ is a family of compact subsets of X. Put $K = \{(\alpha_n) \in \Pi_{n\in\mathbb{N}}\Gamma_n : \bigcap_{n\in\mathbb{N}} S_{\alpha_n} \neq \emptyset\}$. Then

(i) $K \subset M$ and $f(K) \subset S \bigcup \{x\}$: Let $a = (\alpha_n) \in K$, then $\bigcap_{n\in\mathbb{N}} S_{\alpha_n} \neq \emptyset$. Pick $y \in \bigcap_{n\in\mathbb{N}} S_{\alpha_n}$, then $y \in \bigcap_{n\in\mathbb{N}} P_{\alpha_n}$. Note that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a network at y in X if and only if $y \in \bigcap_{n\in\mathbb{N}} P_{\alpha_n}$. So $a \in M$ and $f(a) = y \in S \bigcup \{x\}$. This proves That $K \subset M$ and $f(K) \subset S \bigcup \{x\}$.

(ii) $S \bigcup \{x\} \subset f(K)$: Let $y \in S \bigcup \{x\}$. For every $n \in \mathbb{N}$, pick $\alpha_n \in \Gamma_n$ such that $y \in S_{\alpha_n}$. Put $a = (\alpha_n)$, then $a \in K$ and $f(a) = y$. This proves That $S \bigcup \{x\} \subset f(K)$.

(iii) K is a compact subset of M: Since $K \subset M$ and $\Pi_{n\in\mathbb{N}}\Gamma_n$ is a compact subset of $\Pi_{n\in\mathbb{N}}\Lambda_n$. We only need to prove that K is a closed subset of $\Pi_{n\in\mathbb{N}}\Gamma_n$. It is clear that $K \subset \Pi_{n\in\mathbb{N}}\Gamma_n$. Let $a = (\alpha_n) \in \Pi_{n\in\mathbb{N}}\Gamma_n - K$. Then $\bigcap_{n\in\mathbb{N}} S_{\alpha_n} = \emptyset$. There exists $n_0 \in \mathbb{N}$ such that $\bigcap_{n\leq n_0} S_{\alpha_n} = \emptyset$. Put $W = \{(\beta_n) \in \Pi_{n\in\mathbb{N}}\Gamma_n : \beta_n = \alpha_n\ for\ n \leq n_0\}$. Then W is open in $\Pi_{n\in\mathbb{N}}\Gamma_n$ and $a \in W$. It is easy to see that $W \bigcap K = \emptyset$. So K is a closed subset of $\Pi_{n\in\mathbb{N}}\Gamma_n$.

By the above (i), (ii) and (iii), f is a sequence-covering mapping.

By the above, X is a sequence-covering, π-image of a metric space. $\square$

Lemma 2.5. *Let $\mathcal{P}$ be a point-countable cover of a space X. Then $\mathcal{P}$ is an fcs-cover if and only if $\mathcal{P}$ is a cs^*-cover.*

Proof. Necessity holds by Definition 2.2. We only need to prove sufficiency.

Let $\mathcal{P}$ be a point-countable cs^*-cover of X. Let $S = \{x_n\}$ be a sequence converging to x in X. Since $\mathcal{P}$ is point-countable, put $(\mathcal{P})_x = \{P_n : n \in \mathbb{N}\}$. Then S is eventually in $\bigcup_{n\le k} P_n$ for some $k \in \mathbb{N}$. If not, then for any $k \in \mathbb{N}$, S is not eventually in $\bigcup_{n\le k} P_n$. So, for every $k \in \mathbb{N}$, there exists $x_{n_k} \in S - \bigcup_{n\le k} P_n$. We may assume $n_1 < n_2 < \cdots < n_{k-1} < n_k < n_{k+1} < \cdots$. Put $S' = \{x_{n_k}\}$, then S' is a sequence converging to x in X. Since $\mathcal{P}$ is a cs^*-cover, there exists $m \in \mathbb{N}$ and a subsequence S'' of S' such that S'' is eventually in P_m. This contradicts the construction of S'. □

Recall a mapping $f\colon X \longrightarrow Y$ is an s-mapping, if $f^{-1}(y)$ is a separable subset of X for every $y \in Y$. Combining [13, Theorem 3.3.12] and [19, Lemma 2.2(2)], we have the following corollary.

Corollary 2.6. *Let X be a space. Then the following are equivalent.*

(1) X is a sequence-covering, s and π-image of a metric space.

(2) X is a sequentially-quotient, s and π-image of a metric space.

(3) X has a σ-point-countable strong network consisting of fcs-covers.

(4) X has a σ-point-countable strong network consisting of cs^*-covers.

Proof. (1) $\Longrightarrow$ (2): it is clear.

(2) $\Longrightarrow$ (4): It holds by [13, Theorem 3.3.12].

(4) $\Longrightarrow$(1): It holds by [19, Lemma 2.2(2)].

(3) $\Longleftrightarrow$ (4): It holds by Lemma 2.5. □

3. π-Images of Separable Metric Spaces

Now we discuss sequence-covering (resp. sequentially-quotient), π-images of separable metric spaces.

Definition 3.1. Let X be a space, and let $x \in X$. A subset P of X is called a sequential neighborhood of x ([3]) if every sequence $\{x_n\}$ converging to x in X is eventually in P.

Definition 3.2. Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a cover of a space X. $\mathcal{P}$ is called an sn-network of X ([14]), if $\mathcal{P}_x$ satisfies the following (a),(b) and (c) for every $x \in X$, where $\mathcal{P}_x$ is called an sn-network at x in X.

(a) $\mathcal{P}_x$ is a network at x in X;

(b) if $P_1, P_2 \in \mathcal{P}_x$, then $P \subset P_1 \bigcap P_2$ for some $P \in \mathcal{P}_x$;

(c) every element of $\mathcal{P}_x$ is a sequential neighborhood of x.

Remark 3.3. In [12], a sequential neighborhood of x and an sn-network is called a sequence barrier at x and a universal cs-network respectively.

Theorem 3.4. *For a space X, the following are equivalent.*

(1) X is a sequence-covering, π-image of a separable metric space;

(2) X is a sequentially-quotient, π-image of a separable metric space;

(3) X has a σ-strong network consisting of countable fcs-covers;

(4) X has a σ-strong network consisting of countable cs^*-covers.

Proof. The proofs of (1)$\Longleftrightarrow$(3) and (2)$\Longleftrightarrow$(4) are as the proof of Theorem 2.4. (3)$\Longleftrightarrow$(4) from Lemma 2.5. □

Ge proved that for a regular space X, conditions in Theorem 3.4 are equivalent to that X has a countable sn-network ([4]). The following example shows that "regular" can not be omitted here.

Example 3.5. A space with a countable sn-network is not a sequentially-quotient, π-image of a metric space.

Proof. Let R be the set of all real numbers, and let τ be the Euclidean topology on R. Put $X = R$ with the topology $\tau^* = \{\{x\} \bigcup (D \bigcap U) : x \in U \in \tau\}$, where D is the set of all irrational numbers. That is, X is the pointed irrational extension of R. Then X is Hausdorff, non-regular, and has a countable base ([17, Example 69]), so X has a countable sn-network. Lin showed that X is not a symmetric space ([13, Example 3.13(5)]), so X is not a quotient, π-image of a metric space ([18]). Note that every sequentially-quotient mapping onto a first countable space is quotient ([1]). Thus X is not a sequentially-quotient, π-image of a metric space. □

However, by the proofs of [14, Theorem 4.6 (3)$\Longrightarrow$(2)] and [4, Theorem 2.7(3)$\Longrightarrow$(1)], we have the following results without requiring the regularity of the spaces involved.

Proposition 3.6. *For a space X, the following are true.*

(1) If X is a sequentially-quotient, π-image of a separable metric space, then X has a countable sn-network.

(2) If X has a countable closed sn-network, then X is a compact-covering, compact image of a separable metric space.

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