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ON THE UNIFORM PERFECTNESS OF GROUPS OF BUNDLE HOMEOMORPHISMS

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Abstract. Groups of homeomorphisms related to locally trivial bundles are studied. It is shown that these groups are perfect. Moreover if the homeomorphism isotopy group of the base is bounded then the bundle homeomorphism group of the total space is uniformly perfect.

1. Introduction

Let M be a topological metrizable (and so paracompact) and second countable manifold, let $\mathrm{Homeo}(M)$ denote the group of all homeomorphisms of M and let $\mathrm{Homeo}_c(M)$ be the subgroup of all compactly supported elements. Next the symbol $\mathcal{H}(M)$ (resp. $\mathcal{H}_c(M)$) stands for the subgroup of all elements of $\mathrm{Homeo}(M)$ that can be joined to the identity by an isotopy (resp. a compactly supported isotopy) in $\mathrm{Homeo}(M)$. Recall that a group G is called perfect if it is equal to its commutator subgroup, i.e. any element of G can be expressed as a product of commutators $[f,g]=fgf^{-1}g^{-1}$, where $f,g\in G$. Then we have the following theorem which essentially follows from the results of Mather [10], and Edwards and Kirby [3].

Theorem 1.1 ([7, Theorem 1.1, Cor. 1.3])**.** *Assume that either M is compact (possibly with boundary), or M is open and admits a compact exhaustion. Then the group $\mathcal{H}_c(M)$ is perfect. Moreover, for M connected, $\mathcal{H}_c(M)$ is simple if and only if $\partial_M=\emptyset$.*

As a topological group, $\mathrm{Homeo}(M)$ will be endowed with the Whitney (or graph) topology. If M is compact this topology coincides with the compact-open topology.

From now on we will assume that that M, B and F are topological metrizable and second countable manifolds and $\pi\colon M\to B$ is a locally trivial bundle with the standard fiber F. Denote by $\mathrm{Homeo}(M,\pi)$ (resp. $\mathrm{Homeo}_c(M,\pi)$) the totality of fiber preserving homeomorphism (resp. with compact support). Next denote by $\mathrm{Homeo}_\pi(M)$ the group of all bundle homeomorphisms of π. It is clear that π induces the homomorphism $P\colon\ \mathrm{Homeo}_\pi(M)\to\mathrm{Homeo}(B)$ given by $P(f)(\pi(x))=\pi(f(x))$.

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Let $\mathrm{Homeo}_{\pi,c}(M)$ be the subgroup of $\mathrm{Homeo}_{\pi}(M)$ of all transversely compactly supported bundle homeomorphisms of M. That is, $f \in \mathrm{Homeo}_{\pi,c}(M)$ if and only if f sends each fiber onto another fiber and $\pi(\mathrm{supp}(f))$ is compact. Then $\mathrm{Homeo}_c(M,\pi)$ is a normal subgroup of $\mathrm{Homeo}_{\pi,c}(M)$. The symbol $\mathcal{H}_{\pi,c}(M)$ (resp. $\mathcal{H}_c(M,\pi)$) stands for the group of all homeomorphisms from $\mathrm{Homeo}_{\pi,c}(M)$ (resp. $\mathrm{Homeo}_c(M,\pi)$) that can be joined to the identity by an isotopy in $\mathrm{Homeo}_{\pi,c}(M)$ (resp. $\mathrm{Homeo}_c(M,\pi)$). It follows the existence of the homomorphism $P\colon \mathcal{H}_{\pi,c}(M) \to \mathcal{H}_c(M,\pi)$.

Theorem 1.2. *Let $\pi\colon M \to B$ be a locally trivial bundle with the standard fiber F. If F is closed or is the interior of a compact manifold with boundary then the group $\mathcal{H}_c(M,\pi)$ is perfect. Moreover, if F is closed then $\mathcal{H}_{\pi,c}(M)$ is perfect too.*

Clearly these groups are not simple.

For a topological group $\mathcal{G}$ by $\mathcal{PG}$ we denote the isotopy or path group of $\mathcal{G}$, that is the totality of continuous paths $f\colon I \to \mathcal{G}$ with $f(0) = e$, $I = [0,1]$.

A subgroup $\mathcal{G} \leq \mathrm{Homeo}(M)$ is called *fragmentable* if each element of $\mathcal{G}$ can be written as a product of homeomorphisms from $\mathcal{G}$ supported in open balls. Next $\mathcal{G}$ is said to be *path fragmentable* if the path group $\mathcal{PG}$ is fragmentable. Observe that the group $\mathcal{H}_c(M)$ is path fragmentable (and so fragmentable) due to Theorem 3.3 below.

Recall that a group is called *bounded* if it is bounded with respect to any bi-invariant metric on it. Next a group G is *uniformly perfect* if any element can be expressed as a product of a bounded number of commutators. Clearly any bounded and perfect group is uniformly perfect. See Section 2 for more details.

The main result is the following

Theorem 1.3. *Assume that $\pi\colon M \to B$ is a locally trivial bundle with the standard fiber F closed. Then we have:*

(1) *If $\mathcal{H}_{\pi,c}(M)$ is uniformly perfect then $\mathcal{H}_c(B)$ is also uniformly perfect.*

(2) *If the fragmentation norm on the group $\mathcal{PH}_c(B)$ is bounded, then $\mathcal{H}_{\pi,c}(M)$ is uniformly perfect and*

$$\mathrm{cld}_{\mathcal{H}_{\pi,c}(M)} \leq \mathrm{fd}_{\mathcal{PH}_c(B)} + 2(n+1)\,,$$

where $n = \dim B$ and cld *(resp.* fd*) is the commutator length diameter (resp. fragmentation diameter), cf. Section 2.*

The problem of the algebraic structure of homeomorphism groups, especially the boundedness and uniform perfectness of them, has drawn much attention. It has been studied among others in [1], [4], [5], [6], [7], [8], [9], [11], [12], [14], [15], [16] (see also references therein).

Throughout all manifolds are topological, second countable and metrizable. The symbol of composition in homeomorphism groups will be omitted.

2. Conjugation-invariant norms

The notion of boundedness can be expressed in terms of a conjugation-invariant norms. A *conjugation-invariant norm* on a group G is a function $\nu\colon G \to [0,\infty)$ which satisfies the following conditions. For any g, $h \in G$

(1) $\nu(g) > 0$ if and only if $g \neq e$,

(2) $\nu(g^{-1}) = \nu(g)$,

(3) $\nu(gh) \leq \nu(g) + \nu(h)$,

(4) $\nu(hgh^{-1}) = \nu(g)$.

It is easily seen that G is bounded if and only if any conjugation-invariant norm on G is bounded. By the diameter of ν we mean $\sup_{g\in G} \nu(g)$.

Let G be a group and let $[G,G]$ be its commutator subgroup. For $g \in [G,G]$ the symbol $\mathrm{cl}_G(g)$ stands for the least k such that g is written as a product of k commutators and is called the commutator length of g. Observe that the commutator length cl_G is a conjugation-invariant norm on $[G,G]$. In particular, if G is a perfect group then cl_G is a conjugation-invariant norm on G. For any perfect group G denote by cld_G the commutator length diameter of G, $\mathrm{cld}_G := \sup\limits_{g\in G} \mathrm{cl}_G(g)$. Then G is uniformly perfect iff $\mathrm{cld}_G < \infty$.

Another example of conjugation-invariant norm is the following. Let $\mathcal{G}$ be a subgroup of $\mathrm{Homeo}(M)$ and assume that $\mathcal{G}$ is fragmentable. For $h \in \mathcal{G}$, $h \neq \mathrm{id}$, we define the *fragmentation norm* $\mathrm{frag}_{\mathcal{G}}(h)$ to be the smallest integer $r > 0$ such that $h = h_1 \dots h_r$ with $\mathrm{supp}(h_i) \subset U_i$ for all i, where U_i is a ball. By definition $\mathrm{frag}_{\mathcal{G}}(\mathrm{id}) = 0$. Next by $\mathrm{fd}_{\mathcal{G}}$ we denote the fragmentation diameter of $\mathcal{G}$, $\mathrm{fd}_{\mathcal{G}} := \sup_{h\in\mathcal{G}} \mathrm{frag}_{\mathcal{G}}(h)$.

Recall the notion of displacement of a subgroup. A subgroup K of G is called strongly m-displaceable if there is $g \in G$ such that the subgroups $K, gKg^{-1}, \dots,$ g^mKg^{-m}pairwise commute. Then we say that g m-displaces K.

Proposition 2.1 ([2])**.** *Assume that g m-displaces $K \leq G$ for every $m \geq 1$. Then for any $f \in [K,K]$ we have* $\mathrm{cl}_G(f) \leq 2$*.*

3. Locally continuously fragmentable groups

The following type of fragmentations is important when studying groups of homeomorphisms.

Definition 3.1. A subgroup $\mathcal{G} \leq \mathrm{Homeo}(M)$ is called *locally continuously fragmentable* with respect to a finite open cover $\{U_i\}_{i=1}^d$ if there exist a neighborhood $\mathcal{N}$ of $\mathrm{id} \in \mathcal{G}$ and continuous mappings $\sigma_i : \mathcal{N} \to \mathcal{G}$, $i = 1, \dots, d$, such that $\sigma_i(\mathrm{id}) = \mathrm{id}$ and for all $f \in \mathcal{N}$ one has

$$f = \sigma_1(f) \cdots \sigma_d(f)\,, \quad \mathrm{supp}(\sigma_i(f)) \subset U_i, \forall i\,.$$

Moreover, we assume that each $\mathrm{supp}(\sigma_i(f))$ is compact whenever $f \in \mathrm{Homeo}_c(M)$. If $\mathcal{N} = \mathcal{G}$ then $\mathcal{G}$ is called *globally continuously fragmentable.*

Clearly, if $\mathcal{G}$ is connected and locally continuously fragmentable then it is fragmentable. Observe that Def. 3.1 can also be formulated for $\mathcal{PG}$ rather than $\mathcal{G}$, where $\mathcal{G} \leq \mathcal{H}(M)$.

For a manifold X and a subgroup $\mathcal{G} \leq \mathcal{H}(M)$, let $\mathcal{C}(X, \mathcal{G})$ stand for the group of all continuous maps $X \to \mathcal{G}$ with the pointwise multiplication and the compact-open topology. For $f \in \mathcal{C}(X, \mathcal{G})$ we define $\mathrm{supp}(f) = \bigcup_{x \in X} \mathrm{supp}(f^x)$, where $f^x : p \in M \mapsto f(x)(p) \in M$. Then Def. 3.1 extends obviously for $\mathcal{C}(X, \mathcal{G})$. It is easy to check that if $\mathcal{G}$ is a topological group then $\mathcal{C}(X, \mathcal{G})$ is also a topological group.

Proposition 3.2. *If X is compact and $\mathcal{G}$ is locally continuously fragmentable with respect to $\{U_i\}_{i=1}^d$, then so is $\mathcal{C}(X, \mathcal{G})$.*

Proof. Let $\sigma_i : \mathcal{N} \to \mathcal{G}$, $i = 1, \ldots, d$, be as in Def. 3.1. Define $\hat{\mathcal{N}} = \{f \in \mathcal{C}(X, \mathcal{G}) : f(X) \subset \mathcal{N}\}$ and continuous maps $\sigma_i^{\mathcal{C}} : \hat{\mathcal{N}} \to \mathcal{C}(X, \mathcal{G})$ by the formulae $\sigma_i^{\mathcal{C}}(f)(x) = \sigma_i(f^x)$, where $f \in \mathcal{C}(X, \mathcal{G})$, $x \in X$. It follows that $\mathrm{supp}(\sigma_i^{\mathcal{C}}(f)^x) \subset U_i$ for all i and x. Consequently we have $\mathrm{supp}(\sigma_i^{\mathcal{C}}(f)) \subset U_i$. □

The results of this paper depend essentially on the deformation properties for the spaces of imbeddings obtained by Edwards and Kirby in [3]. See also Siebenmann [17]. All manifolds are assumed to be metrizable and second countable (i.e. have at most countably many connected components).

A *ball* U in an n-dimensional manifold M is a relatively compact, open ball imbedded with its closure in M. For M closed, let $d = d_M$ is the smallest integer such that $M = \bigcup_{i=1}^d U_i$ where U_i is a ball if M is connected (or U_i is the union of disjoint open balls, each ball lies in a different connected component, if M is not connected) such that $\mathrm{cl}(U_i) \neq M$ for each i. Then $d \leq n+1$. Next if M is an open manifold, then M admits an open cover $\{U_i\}_{i=1}^{n+1}$ (see [13]), called a Palais cover, such that each U_i is the union of a countable, locally finite family of balls with pairwise disjoint closures. In each case such a cover will be called *related* to M.

Theorem 3.3. *Assume that M is closed or M is the interior of a compact manifold. Then $\mathcal{H}_c(M)$ is a locally continuously fragmentable group (Def. 3.1) with respect to any cover $\{U_i\}_{i=1}^d$, $d \leq n+1$, related to M. Moreover the isotopy group $\mathcal{PH}_c(M)$ is also locally continuously fragmentable with respect to $\{U_i\}_{i=1}^d$.*

For the proof see Prop. 3.2 [16] and the proof of Theorem 1.3 in [16]. The second claim follows from Prop. 3.2 above.

Proposition 3.4. *If $\pi : M \to B$ be a locally trivial bundle then $\mathcal{PH}_c(M, \pi)$ is globally continuously fragmentable with respect to $\{\pi^{-1}(U_i)\}$, where $\{U_i\}$ is a cover related to B. Next if F is closed or is the interior of a compact manifold with boundary, and $\{V_j\}$ is a cover related to F, then $\mathcal{PH}_c(M, \pi)$ is locally continuously fragmentable with respect to $\{U_i \times V_j)\}_{i,j}$.*

Proof. To show the first claim, suppose that $\{\lambda_i\}_{i=1}^d$ is a partition of unity subordinate to $\{U_i\}_{i=1}^d$, a cover related to B. Let $g = \{g^t\} \in \mathcal{PH}_c(M, \pi)$.

For $p \in M$ put $h_1^t(p) = g^{\lambda_1(\pi(p))t}(p)$. Next

$$h_2^t(p) = \left((g^{\lambda_1(\pi(p)t)})^{-1} g^{(\lambda_1+\lambda_2)(\pi(p))t}\right)(p)\,.$$

In general, for $3 \le i \le d$ we define

$$h_i^t(p) = \big((g^{(\lambda_1+\cdots\lambda_{i-1})(\pi(p))t)})^{-1} g^{(\lambda_1+\cdots+\lambda_i)(\pi(p))t}\big)(p)\,.$$

Then $\mathrm{supp}(h_i^t) \subset \pi^{-1}(U_i)$ for all i and t, and $g^t = h_1^t \dots h_d^t$. Moreover, the maps $\sigma_i\colon g \mapsto h_i = \{h_i^t\}$ are continuous.

In order to show the second assertion let $g \in \mathcal{PH}_c(M,\pi)$ and $\{U_i\}_{i=1}^d$ be related to B so that π trivializes over each U_i. We suppose that g is so small that for each h_i^t as above we have that $h_i^t|_{\pi^{-1}(x)} \in \mathcal{N}$, where $\mathcal{N}$ is as in Def. 3.1, for all $x \in U_i$, $i = 1, \dots, d$. Then we apply Theorem 3.3 to the family $h_i^t|_{\pi(x)}$, $x \in U_i$, in a fiberwise fashion, with respect to a cover $\{U_i \times V_j\}_j$, where $\{V_j\}$ is related to F. It follows that $\mathcal{PH}_c(M,\pi)$ is locally continuously fragmentable with respect to $\{U_i \times V_j\}_{i,j}$. □

The following basic lemma for homeomorphisms is no longer true in the C^1 category.

Lemma 3.5 ([10], [16]). *Let W and V be balls such that $\mathrm{cl}(W) \subset V$. Then there exist $\phi \in \mathcal{H}_c(V)$ and a continuous map $S\colon \mathcal{H}_c(W) \to \mathcal{H}_c(V)$ such that $g = [S(g), \phi]$ for all $g \in \mathcal{H}_c(W)$.*

We also need the following

Proposition 3.6. *If $\pi\colon M \to B$ is a locally trivial bundle with the standard fiber F closed, then the homomorphism $P\colon \mathcal{H}_{\pi,c}(M) \to \mathcal{H}_c(B)$ is surjective. Furthermore, the induced map for isotopies $P\colon \mathcal{PH}_{\pi,c}(M) \to \mathcal{PH}_c(B)$ is also surjective.*

Proof. Obviously, the second assertion implies the first. To show the second, let $h \in \mathcal{PH}_c(B)$. From Theorem 3.3 it follows that $h = h_1 \dots h_p$ with all $h_i \in \mathcal{PH}_c(B)$ supported balls. Consequently, each h_i can be lifted by means of a trivialization of π to an isotopy $\tilde{h}_i \in \mathcal{PH}_{\pi,c}(M)$ such that $P(\tilde{h}_i) = h_i$. Thus $P(\tilde{h}) = h$, where $\tilde{h} = \tilde{h}_1 \dots \tilde{h}_p$. □

Proof of Theorem 1.2. First we prove that $\mathcal{H}_c(M,\pi)$ is perfect. According to Prop. 3.4 it suffices to assume that $M = U \times V$, where U (resp. V) is a ball in B (resp. in F) and π is the projection $U \times V \to U$. If $f \in \mathcal{H}_c(U \times V, \pi)$ then $\mathrm{supp}(f) \subset U \times W$ such that $\mathrm{cl}(W) \subset V$. In view of Lemma 3.5 there are $\phi \in \mathcal{H}_c(V)$ and a continuous map $S\colon \mathcal{H}_c(W) \to \mathcal{H}_c(V)$ such that $g = [S(g), \phi]$ for all $g \in \mathcal{H}_c(W)$.

Consider $\tilde{S}\colon \mathcal{H}_c(U \times W) \to \mathcal{H}_c(U \times V)$ given by $\tilde{S}(\tilde{g})(x,y) = (x, S(\tilde{g}|_{\pi^{-1}(x)})(y))$ for all $\tilde{g} \in \mathcal{H}_c(U \times W)$. Then we get $f = [\tilde{S}(f), \mathrm{id} \times \phi]$. It remains to modify $\mathrm{id} \times \phi$ to be compactly supported. Namely, if $\phi = \bar{\phi}^1$ with $\bar{\phi} \in \mathcal{PH}_c(V)$ and $\lambda\colon B \to [0,1]$ is a compactly supported bump function satisfying $\lambda|_{\pi(\mathrm{supp}(f))} = 1$, then we may use $\hat{\phi}$ given by $\hat{\phi}(x,y) = (x, \bar{\phi}^{\lambda(x)}(y))$ instead of $\mathrm{id} \times \phi$.

To show that $\mathcal{H}_{\pi,c}(M)$ is perfect, let $f \in \mathcal{H}_{\pi,c}(M)$. Take $\bar{f} \in \mathcal{PH}_{\pi,c}(M)$ such that $\bar{f}^1 = f$. According to Theorem 3.3 we have a decomposition of isotopies $P(\bar{f}) = h_1 \dots h_r$, where each h_i is supported in a ball in B, say U_i (or a union of locally finite family of balls with pairwise disjoint closures). In view of Prop. 3.6 there exist lifts $\bar{h}_1, \dots, \bar{h}_r \in \mathcal{PH}_{\pi,c}(M)$ of $h_1, \dots, h_r$, resp. If $\bar{h} := \bar{h}_1 \dots \bar{h}_r$ then

$\bar{g} = \bar{f}\bar{h}^{-1}$ lies in $\ker(P)$. Consequently, $\bar{g}^1$ belongs to $\mathcal{H}_c(M,\pi)$ and so is a product of commutators due to the first part. On the other hand, each $\bar{h}_i^1$ can be viewed as an element of $\mathcal{H}_c(U_i)$ if we use a local trivialization over U_i. Therefore, due to Lemma 3.5, each $\bar{h}_i^1$ is a commutator. Thus $f = \bar{f}^1 = \bar{g}^1\bar{h}^1$ is a product of commutators and $\mathcal{H}_{\pi,c}(M)$ is perfect. □

4. Proof of Theorem 1.3

(1) It follows from Prop. 3.6.

(2) Let $f \in \mathcal{H}_{\pi,c}(M)$ and let $\bar{f} \in \mathcal{PH}_{\pi,c}(M)$ such that $\bar{f}^1 = f$. We have $P(\bar{f}) \in \mathcal{PH}_c(B)$ and we may use Theorem 3.3 to get a fragmentation of isotopies

$$P(\bar{f}) = h_1 \dots h_p\,,$$

where each isotopy $h_i \in \mathcal{PH}_c(U_i)$, where U_i is a ball for all i, and where p is bounded according to the assumption. In view of Prop. 3.6 we define $\bar{h}_i \in \mathcal{PH}_{\pi,c}(\pi^{-1}(U_i))$, the lifts of h_i, and we put

$$\bar{h} = \bar{h}_1 \dots \bar{h}_p \quad \text{and} \quad \bar{g} = \bar{f}\bar{h}^{-1}\,.$$

Since $P(\bar{h}) = P(\bar{f})$, it follows that $\bar{g}$ is an isotopy in $\ker(P)$ joining $g = \bar{g}^1$ to the identity.

By using local trivialization of π, each $\bar{h}_i$ can be regarded as an element of $\mathcal{PH}_c(U_i)$. Then due to Lemma 3.5 each $\bar{h}_i^1$ can be written as a commutator.

Now in view of Prop. 3.4 $g = \bar{g}^1$ can be written as a product of at most $n+1$ factors,

$$g = g_1 \dots g_{n+1}\,, \quad \operatorname{supp}(g_i) \subset \pi^{-1}(U_i)$$

for all i, where U_i is a ball provided B is closed. If B is open then U_i is a finite union of balls with disjoint closures. In both cases each g_i can be expressed as a product of at most two commutators. In fact, for every U_i there exists $\phi_i \in \mathcal{H}_{\pi,c}(M)$ such that the family $\{\phi_i^k(\pi^{-1}(U_i))\}_{k\in\mathbb{N}_0}$ is pairwise disjoint, and we apply Theorem 1.2 and Prop. 2.1. Observe that the reasoning from [10] does not apply in this case.

Consequently, f can be expressed as a product of $p + 2(n+1)$ commutators. Therefore $\mathcal{H}_{\pi,c}(M)$ is uniformly perfect and

$$\operatorname{cld}_{\mathcal{H}_{\pi,c}(M)} \le \operatorname{fd}_{\mathcal{PH}_c(B)} + 2(n+1)\,,$$

as required.

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