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CORRECTIONS TO “THE MAXIMAL FUNCTION ON VARIABLE L^p SPACES”

D. Cruz-Uribe, A. Fiorenza, and C. J. Neugebauer

Trinity College, Department of Mathematics
Hartford, CT 06106-3100, U.S.A.; david.cruzuribe@mail.trincoll.edu

Universitá di Napoli, Dipartimento di Costruzioni e Metodi Matematici in Architettura
Via Monteoliveto, 3, I-80134 Napoli, Italy; fiorenza@unina.it
and Consiglio Nazionale delle Ricerche, Istituto per le Applicazioni del Calcolo
“Mauro Picone” – sezione di Napoli
via Pietro Castellino, 111, I-80131 Napoli, Italy

Purdue University, Department of Mathematics
West Lafayette, IN 47907-1395, U.S.A.; neug@math.purdue.edu

We have discovered a small but significant error in our recent paper, *The maximal function on variable L^p spaces* (Ann. Acad. Sci. Fenn. Math. 28 (2003), 223–238). The main result—sufficient conditions on the exponent function $p(\,\cdot\,)$ for the Hardy–Littlewood maximal operator to be bounded on $L^{p(\,\cdot\,)}$—remains true. However, the proof was improperly condensed and important details are missing. In two places (the proof of Case 2 of Lemma 2.3 and the proof of Lemma 2.5) we omitted the proof, claiming that it was identical to the proof of Case 1 of Lemma 2.3. The missing arguments are very similar, but are not identical, and require a more general version of a key lemma. In this note we give the details of the correct argument. We presume that the reader is familiar with the contents and notation of our original paper.

At the heart of our correction is the following lemma which replaces Lemma 2.2.

Lemma 1. *Given a set G and two non-negative functions $r(\,\cdot\,)$ and $s(\,\cdot\,)$, suppose that for each $y \in G$,*

$$0 \le s(y) - r(y) \le \frac{C}{\log\bigl(e + |z(y)|\bigr)},$$

where $z\colon G \to \mathbf{R}^n$ is measurable. Then for every function f,

$$\int_G |f(y)|^{r(y)}\,dy \le C\int_G |f(y)|^{s(y)}\,dy + \int_G \alpha(z(y))^{r_*(G)}\,dy,$$

where $\alpha(y) = (e + |y|)^{-n}$.

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When $z(y) = y$, this reduces to Lemma 2.2. The proof is identical to the proof of Lemma 2.2, except that in the proof the set G^α is redefined as

$$G^\alpha = \{x \in G : |f(x)| \geq \alpha\big(z(x)\big)\}.$$

In the proof of Lemma 2.3, Case 1, we are given a point x and a ball B containing x whose radius r is such that $r < |x|/4$. In this case, inequality (1.4) yields inequality (2.2),

$$0 \leq \bar{p}(y) - \bar{p}_*(B_\Omega) \leq \frac{C}{\log(e + |y|)}, \qquad y \in B_\Omega,$$

and this inequality provides the condition we use to apply the lemma above with $z(y) = y$.

Our mistake was in assuming that inequality (2.2) held when it did not. In Case 2 of the proof of Lemma 2.3, we are given x, $|x| \leq 1$, and a ball B containing x. Since we are not assuming that the radius of B is bounded by a multiple of $|x|$, (2.2) need not hold. However, since $|x| \leq 1$, for all $y \in B_\Omega$,

$$0 \leq \bar{p}(y) - \bar{p}_*(B_\Omega) \leq p^* - p_* \leq \frac{C}{\log(e + |x|)}.$$

Therefore, we can apply the lemma with $z(y) = x$. Then

$$\begin{aligned}\left(\frac{1}{|B|}\int_{B_\Omega} f(y)\,dy\right)^{p(x)} &\leq \left(\frac{1}{|B|}\int_{B_\Omega} f(y)^{\bar{p}_*(B_\Omega)}\,dy\right)^{p(x)/\bar{p}_*(B_\Omega)} \\ &\leq \Bigg(\frac{C}{|B|}\int_{B_\Omega} f(y)^{\bar{p}(y)}\,dy \\ &\qquad + \frac{1}{|B|}\int_{B_\Omega} \alpha\big(z(y)\big)^{\bar{p}_*(B_\Omega)}\,dy\Bigg)^{p(x)/\bar{p}_*(B_\Omega)};\end{aligned}$$

since $z(y) = x$ we now get immediately that

$$\leq \left(\frac{C}{|B|}\int_{B_\Omega} f(y)^{\bar{p}(y)}\,dy + C\alpha(x)^{\bar{p}_*(B_\Omega)}\right)^{p(x)/\bar{p}_*(B_\Omega)}.$$

Now the argument proceeds exactly as in Case 1.

Similarly, in the proof of 2.5, we are given x, $|x| \geq 1$, and a ball B containing x. Again we assumed that inequality (2.2) held when it did not. But, if $y \in B_\Omega \setminus B_{|x|}(0)$, then it follows from inequality (1.4) that

$$0 \leq \bar{p}(y) - \bar{p}_*\big(B_\Omega \setminus B_{|x|}(0)\big) \leq \bar{p}^*\big(B_\Omega \setminus B_{|x|}(0)\big) - \bar{p}_*\big(B_\Omega \setminus B_{|x|}(0)\big) \leq \frac{C}{\log(e + |x|)}.$$

Therefore, to estimate

$$\left(\frac{1}{|B|}\int_{B_\Omega \setminus B_{|x|}(0)} f(y)\, dy\right)^{p(x)}$$

we can argue exactly as we did above in the new proof of Case 2 in Lemma 2.3, and then complete the proof as in Case 1.

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