Fields, rings, and modules
Exercise sheet 4

https://www.maths.tcd.ie/~mascotn/teaching/2020/MAU22102/index.html

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Exercise 1 *A non-free module over a non-commutative ring*

Let $M_2 = M_2(\mathbb{R})$ be the ring of 2×2 matrices with real entries, and let $V = \mathbb{R}^2$ be the space of column vectors of size 2 with real entries.

1. Prove that the natural multiplication $M_2 \times V \longrightarrow V$ gives V the structure of an M_2-module (so that the elements of V are the "vectors" and the elements of M_2 are the "scalars").
2. Find a generating set for the M_2-module V containing as few elements as possible.
3. Prove that V is **not** a free M_2-module.

   Hint: Consider the dimensions of the underlying $\mathbb{R}$*-vector spaces.*

Exercise 2 *Finitely generated Abelian groups*

1. Let G be the Abelian group with generators g, h and relations $8g + 12h = 6g + 8h = 0$. Perform an SNF computation to determine what G is isomorphic to.
2. Determine $\#G$. Is G cyclic?
3. Find all Abelian groups of order 2020, up to isomorphism.