

Math 261 — Exercise sheet 5

<http://staff.aub.edu.lb/~nm116/teaching/2018/math261/index.html>

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Answers are due for Wednesday 24 October, 11AM.

The use of calculators is allowed.

Exercise 5.1: Some factorizations of polynomials mod p (100 pts)

Let $f(x)$ be the polynomial $x^3 - 8x^2 + 19x + 1$. Factor $f(x)$

1. (25 pts) mod 2,
2. (25 pts) mod 3,
3. (25 pts) mod 7,
4. (25 pts) mod 13.

Make sure that your factorizations are complete, i.e. prove that the factors that you find are irreducible.

The exercises below are not mandatory. They are not worth any points, and are given here for you to practise. The solutions will be made available with the solutions to the other exercises.

Exercise 5.2: More factorizations

Let $f(x)$ be the polynomial $x^3 - 3x^2 - 1$. Factor $f(x)$

1. mod 2,
2. mod 3,
3. mod 5,
4. mod 7.

Make sure that your factorizations are complete, i.e. prove that the factors that you find are irreducible.

Exercise 5.3: Irreducible polynomials over $\mathbb{Z}/2\mathbb{Z}$

1. Find all irreducible polynomials of degree 2 over $\mathbb{Z}/2\mathbb{Z}$.
2. Use the previous question and a Euclidean division to deduce that the polynomial $x^4 + x + 1$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$.