

Math 261 — Exercise sheet 8

<http://staff.aub.edu.lb/~nm116/teaching/2017/math261/index.html>

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Answers are due for Monday 4 December, 11AM.

The use of calculators is allowed.

Exercise 8.1: Continued fraction expansions (30 pts)

1. (15 pts) Express the rational $\frac{261}{101}$ as a continued fraction.
2. (15 pts) Compute the continued fraction expansion of the golden ratio $\Phi = \frac{1 + \sqrt{5}}{2}$, and the 4 first convergents.

Exercise 8.2: A Pell-Fermat equation (40 pts)

1. (20 pts) Compute the continued fraction expansion of $\sqrt{6}$.
2. (10 pts) Use the previous question to find the fundamental solution to the equation $x^2 - 6y^2 = 1$.
3. (10 pts) Use the ring structure of $\mathbb{Z}[\sqrt{6}]$ to find 2 other non-trivial solutions (changing the signs of x and y does not count !)

Exercise 8.3: The battle of Hastings (30 pts)

The battle of Hastings was a major battle in English history. It took place on October 14, 1066.

The following fictional historical text, taken from *Amusement in Mathematics* (H. E. Dudeney, 1917), refers to it:

“The men of Harold stood well together, as their wont was, and formed thirteen squares, with a like number of men in every square thereof. (...) When Harold threw himself into the fray the Saxons were one mighty square of men, shouting the battle cries ‘Ut!’, ‘Olicrosse!’, ‘Godemite!’.”

Use continued fractions to determine how many soldiers this fictional historical text suggests Harold II had at the battle of Hastings.

The exercises below are not mandatory. They are not worth any points, and are given here for you to practise. The solutions will be made available with the solutions to the other exercises.

Exercise 8.4

Let $x \in (0, 1)$ be irrational, and let $[a_0, a_1, \dots, a_n] = p_n/q_n$ ($n \in \mathbb{N}$) be the convergents of the continued fraction expansion of x . Prove that

$$x = \sum_{n=0}^{+\infty} \frac{(-1)^n}{q_n q_{n+1}}.$$

Hint: Where could the $(-1)^n$ come from ?

Exercise 8.5

Redo exercise 8.2, with 6 replaced by 14, 15, 17, and 18.