Math 261 — Exercise sheet 7

http://staff.aub.edu.lb/~nm116/teaching/2017/math261/index.html

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Answers are due for Monday 20 November, 11AM.

The use of calculators is allowed.

Exercise 7.1: Reduction (20 pts)

1. (10 pts) Find a reduced quadratic form equivalent to the form $22x^2-16xy+3y^2$.
2. (10 pts) Are the forms $2x^2+xy+3y^2$ and $2x^2-xy+3y^2$ equivalent ?

Exercise 7.2: Class numbers (30 pts)

Compute the class number $h(D)$ for

1. (15 pts) $D=-116$,
2. (15 pts) $D=-47$.

Exercise 7.3: Primes of the form... (30 pts)

Let $p \in \mathbb{N}$ be prime.

1. (15 pts) Prove that p is of the form x^2+3y^2 (with $x, y \in \mathbb{Z}$) if and only if $p=3$ or $p \equiv 1 \pmod 3$.
2. (15 pts) Prove that p is of the form $x^2+xy+3y^2$ (with $x, y \in \mathbb{Z}$) if and only if $p=11$ or $p \equiv 1, 3, 4, 5$ or $9 \pmod{11}$.

*Note: You are **not** allowed to use the theorem giving the list of D such that $h(D)=1$ in this exercise.*

Exercise 7.4: An easy case of the class number 1 problem (20 pts)

1. (10 pts) Let $n \in \mathbb{N}$ be congruent to 1 or 2 mod 4. Prove that $h(-4n)=1$ if and only if $n<3$.

   Hint: Imagine that you apply the method seen in class to compute $h(-4n)$. What happens when $n \geqslant 3$?

2. (10 pts) Let $n \in \mathbb{N}$. Prove that if $h(-4n+1)=1$, then $n=2$ or n is odd.