

Math 261 — Exercise sheet 3

<http://staff.aub.edu.lb/~nm116/teaching/2017/math261/index.html>

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Answers are due for Monday 02 October, 11AM.

The use of calculators is allowed.

Exercise 3.1: Factorization of polynomials mod p (40 pts)

Let $f(x)$ be the polynomial $x^3 - 3x^2 - 1$. Factor $f(x)$

1. (10 pts) mod 2,
2. (10 pts) mod 3,
3. (10 pts) mod 5,
4. (10 pts) mod 7.

Make sure that your factorizations are complete, i.e. prove that the factors that you find are irreducible.

Exercise 3.2: (20 pts)

Find an integer x such that $x \equiv 12 \pmod{7}$ and $x \equiv 7 \pmod{12}$.

Exercise 3.3: (10 pts)

Compute $\phi(261)$ and $\phi(6000)$.

Exercise 3.4: $\phi(n)$ is always even (30 pts)

Prove that $\phi(n)$ is even for all $n \geq 3$.