442 Tutorial Sheet 6[1]

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1. (3) The equation of state is often written in *adiabatic form*

$$p = (\gamma - 1)\rho \tag{1}$$

   where p is pressure and ρ is density and $0 \leq \gamma \leq 2$ is the *adiabatic index* with $\gamma = 0$ for dust and $\gamma = 4/3$ for radiation. Calculate $\rho(a)$ for general γ. $k = 0$ calculate $a(t)$. Find the age of the universe for $k = 0$ and general γ.

2. (2) In the notation of the previous question, find γ so that the expansion rate is constant. With this value of γ find $a(t)$ for $k = 1$ and $k = -1$.

3. (2) In the same notation, show

$$\dot{\Omega} = (2 - 3\gamma)H\Omega(1 - \Omega) \tag{2}$$

   Define the *logarithmic scale factor* $s = \log a$ and write an equation for $d\Omega/ds$. Notice that this formula gives a clear idea of how Ω behaves.

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