442 Tutorial Sheet 4[1]

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1. (2) This question is revision of Lagrangian mechanics. Derive the Euler-Lagrance equations from $\delta S = 0$ where
$$S = \int L(t, \mathbf{q}, \dot{\mathbf{q}}) dt \tag{1}$$

2. (2) Derive the Euler-Largrange equation
$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{2}$$
from $\delta S = 0$ where
$$S = \int \mathcal{L}(t, \phi, \partial_\mu \phi) d^4x \tag{3}$$

3. (3) Derive the Euler-Largange equation
$$\nabla_\mu \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{4}$$
from $\delta S = 0$ where
$$S = \int \mathcal{L}(t, \phi, \nabla_\mu \phi) \sqrt{g} d^4x \tag{5}$$
Recall that $\nabla_\mu \phi = \partial_\mu \phi$ and that $\sqrt{g} \nabla_\mu X^\mu = \partial_\mu \sqrt{g} X^\mu$.

4. (4) Calculate the energy-momentum tensor
$$T_{\mu\nu} = F_{\lambda\mu} F^\lambda{}_\nu - \frac{1}{4} g_{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \tag{6}$$
for the Maxwell field:
$$S = -\frac{1}{4} \int F^{\mu\nu} F_{\mu\nu} \sqrt{g} d^4x \tag{7}$$
where
$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \tag{8}$$

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