Course 414 2005-06

Sheet 5

Due: after the lecture next Friday

Exercise 1

Find the maximal open set, where the sequence f_n converges compactly (uniformly on every compactum):

(i) $f_n(z) = (z + 1/n)^2$;

(ii) $f_n(z) = z^{n^2}$;

(iii) $f_n(z) = \cos z^{2^n}$.

Exercise 2

Let $\Omega \subset \mathbb{C}$ be bounded open set and $(f_n) \in C(\overline{\Omega}) \cap \mathcal{O}(\Omega)$ be a sequence of functions. Given that (f_n) converges uniformly on $\partial\Omega$, prove that it also converges uniformly on Ω. (Hint. Use the Maximum Principle.)

Exercise 3

Determine the Laurent series expansion of the function f at a and its ring of convergence:

(i) $f(z) = (z^2 + 1)^{-1}$, $a = i$;

(ii) $f(z) = (z - 1)^{-2} \mathsf{Log}\, z$, $a = 1$;

(iii) $f(z) = z(z - 2)^2 \cos \pi z$, $a = 2$;

Exercise 4

Let $f(z) = \sum_{n=-\infty}^{\infty} a_n(z - z_0)^n$ and $g(z) = \sum_{n=-\infty}^{\infty} b_n(z - z_0)^n$ be Laurent series converging in a ring $r < |z - z_0| < R$. Find the formula for the Laurent series expansion of the product fg and show that it converges in the same ring.

Exercise 5

Determine the mutliplicity with which f takes its value at z_0:

(i) $f(z) = e^{z \cos z - z}$, $z_0 = 0$;

(ii) $f(z) = (\mathsf{Log}(\cos z))^2$, $z_0 = 2\pi$.

(iii) $f(z) = (1 + z^2 - e^{z^2})^4$, $z_0 = 0$.