

Course 414 2005-06**S h e e t 3**

Due: after the lecture next Friday

Exercise 1

Let $\gamma: [a, b] \rightarrow \mathbb{C}$ be a piecewise smooth path and consider a reparametrization (i.e. continuous monotonic increasing bijection) $\varphi: [\alpha, \beta] \rightarrow [a, b]$.

- (i) Show that if φ is piecewise smooth, so is $\gamma \circ \varphi$.
- (ii) Give an example of γ piecewise smooth and φ not piecewise smooth such that $\gamma \circ \varphi$ is also not piecewise smooth.

Exercise 2

Evaluate the integral $\int_{\gamma} f(z)dz$ using the given parametrization of γ if $f(z) = (z+2)/z$ and

- (i) $\gamma(t) = 2e^{it}$, $0 \leq t \leq \pi$;
- (ii) $\gamma(t) = 2e^{it}$, $\pi \leq t \leq 2\pi$;
- (ii) $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$.

Exercise 3

Evaluate the integral $\int_{\gamma} f(z)dz$ using antiderivatives if $\gamma(t) = 2e^{it}$, $0 \leq t \leq \pi$, and

- (i) $f(z) = \sin z$;
- (ii) $f(z) = z^n$, $n \in \mathbb{Z}$;

Exercise 4

Let γ be the sum of two line segments connecting -1 with iy and iy with 1 , where y is a fixed parameter.

- (i) Write an explicit parametrization for γ ;
- (ii) For every y , evaluate the integrals $\int_{\gamma} z dz$ and $\int_{\gamma} \bar{z} dz$.

Which of the integrals is independent of y ?