Course 414 2005-06

Sheet 2

Due: after the lecture next Friday

Exercise 1

Let $f\colon \Omega \to \mathbb{C}$ be holomorphic. Define the new function $\bar{f}$ by $\bar{f}(z) := \overline{f(\bar{z})}$. Show that $\bar{f}$ is also holomorhic on the open set $\bar{\Omega} := \{\bar{z} : z \in \Omega\}$.

Exercise 2

Give an example of a maximal open set $\Omega \subset \mathbb{C}$, where the given multiple-valued function has a holomorphic branch:

(i) $z^{1/3}$;

(ii) $\mathsf{log}(z+1)$;

(iii) $\sqrt{e^z}$.

Exercise 3

Using the Cauchy-Riemann equations, show:

(i) if a holomorphic function f satisfies $\mathsf{Re} f = \text{const}$, then $f = \text{const}$.

(ii) if $f = u + iv$ is holomorphic and $a, b \in \mathbb{C} \setminus \{0\}$ are such that $au + bv = \text{const}$, then again $f = \text{const}$.

Exercise 4

Find a polynomial function of x and y that is complex-differentiable at every point of the parabola $y = x^2$ but at no other point of the complex plane.