December 3, 2005 Lecturer Dmitri Zaitsev Michaelmas Term 2005

Course 2E1 2005-06 (SF Engineers & MSISS & MEMS)

S h e e t 8

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Use the method of Lagrange multipliers to find the extreme values (maxima and minima) of the function f subject to the constraint.

(i) $f(x, y) = 2xy$ on the ellipse $x^2 + 9y^2 = 1$;

(ii) $f(x, y) = x^2y$ on the line segment $x + y = 5$, $0 \leq x \leq 5$;

(iii) $f(x, y) = x - 6y + 6$ on the circle $x^2 + y^2 = 4$;

(iv) $f(x, y, z) = x + y + 3z$ on the sphere $x^2 + y^2 + z^2 = 16$.

Exercise 2

Find the absolute maximum and minimum of the function f in the plain region R (use the first derivative test to find the critical points in the interior, the method of Lagrange multipliers for the boundary curves, add the corners to the set of points found and compare the values of f to select the maximal and minimal ones):

(i) $f(x, y) = x^2 + y^2 - 2x - 4y$, R is the triangular region bounded by the x- and y-axes and the line $x + y = 6$;

(ii) $f(x, y) = x + y$, R is the region above the parabola $y = x^2 - 9$ and below the x-axis.