

Course 2E1 2005-06 (SF Engineers & MSISS & MEMS)**S h e e t 14**

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Find the matrix for the linear transformations T defined by the equations

- (i) $w_1 = x_1, \quad w_2 = x_1 + x_2,$
- (ii) $w_1 = 2x - z, \quad w_2 = -y, \quad w_3 = 3x,$
- (iii) $w_1 = x_4, \quad w_2 = x_4 + x_3, \quad w_3 = x_4 + x_3 + x_2, \quad w_4 = x_4 + x_3 + x_2 + x_1,$
and by the formulas
- (v) $T(x_1, x_2) = (-x_1, x_2),$
- (iv) $T(x_1, x_2, x_3) = (-x_2, x_1, x_2 - x_1, 5x_1, -4x_3).$

Exercise 2

Find $T(\mathbf{x}) = A\mathbf{x}$ for the matrix A and the vector \mathbf{x} whenever the product makes sense (i.e. the sizes of A and \mathbf{x} fit together):

- (i) $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$
- (ii) $A = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix},$
- (iii) $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

Exercise 3

Use matrix multiplication to find:

- (i) the reflection of the vector $(2, 3)$ about the y -axis;
- (ii) the orthogonal projection of the vector $(1, -2)$ to the x -axis;
- (iii) the image of the vector $(-1, -1)$ under rotation through the angle $-\frac{\pi}{3}$ about the origin.
- (iv) the image of the vector $(2, 1)$ under rotation through the angle $\frac{\pi}{4}$ about the origin.