Course 2E1 2004-05 (SF Engineers & MSISS & MEMS)

S h e e t 1

Due: in the tutorial sessions Wednesday/Thursday, 20th/21th of October 2004

Exercise 1

Find the domain and range of the following functions and draw their graphs:

(i) $f(x) = -x^2$,
(ii) $f(t) = 1 + t^3$,
(iii) $f(x) = 1/\sqrt{1-x^2}$,
(iv) $f(x) = 2^{-x} + 1$,
(v) $f(t) = \ln(t+1)$,
(vi) $f(t) = \mathsf{sin}(2\pi t) + 2$.

Solution

(i) $y = -x^2$: domain $-\infty < x < \infty$; range $y \leq 0$,
(ii) $y = 1 + t^3$: domain $-\infty < t < \infty$; range $-\infty < y < \infty$,
(iii) $y = 1/\sqrt{1-x^2}$: domain $-1 < x < 1$,

To determine the range, find for which y the equation $y = 1/\sqrt{1-x^2}$ has solution in x:

$$y = 1/\sqrt{1-x^2} \iff y > 0, y^2(1-x^2) = 1 \iff y > 0, x^2y^2 = y^2 - 1$$

$$\iff y > 0, x = \pm\sqrt{\frac{y^2-1}{y^2}}.$$

Hence the range is $y > 1$.

(iv) $y = 2^{-x} + 1$: domain $-\infty < x < \infty$; range $y > 1$,
(v) $y = \ln(t+1)$: domain $t > -1$; range $-\infty < y < \infty$,
(vi) $y = \mathsf{sin}(2\pi t) + 2$: domain $-\infty < t < \infty$; range $1 < y < 3$.

Exercise 2

Find the limits:

$$\text{(i)} \lim_{x\to -1}(x^2 - 3^x), \quad \text{(ii)} \lim_{x\to 1}\frac{x^2+x-2}{x-1}, \quad \text{(iii)} \lim_{x\to 0^-} e^{\frac{1}{x}}.$$

Solution (i) $1 - \frac{1}{3}$; (ii) $\lim_{x \to 1} \frac{x^2+x-2}{x-1} = \lim_{x \to 1}(x+2) = 3$; (iii) 0.

Exercise 3

Calculate derivatives of functions in Exercise 1.

Solution

(i) $f(x) = -x^2$, $f'(x) = -2x$.

(ii) $f(t) = 1 + t^3$, $f'(t) = 3t^2$.

(iii) $f(x) = 1/\sqrt{1-x^2} = (1-x^2)^{-1/2}$, $f'(x) = (-2x) \cdot (-\frac{1}{2}(1-x^2)^{-3/2})$.

(iv) $f(x) = 2^{-x} + 1$, $f'(x) = -2^{-x} \ln 2$.

(v) $f(t) = \ln(t+1)$, $f'(t) = \frac{1}{t+1}$.

(vi) $f(t) = \mathsf{sin}(2\pi t) + 2$, $f'(t) = 2\pi \mathsf{cos}(2\pi t)$.