November 26, 2014 Lecturer Dmitri Zaitsev Michaelmas Term 2014

Course 2325 Complex Analysis I

Sheet 3

Due: at the end of the lecture on Wednesday next week

Exercise 1

Let γ be the sum of two line segments connecting -1 with iy and iy with 1, where y is a fixed parameter.

(i) Write an explicit parametrization for γ;

(ii) For every y, evaluate the integrals $\int_\gamma z\,dz$ and $\int_\gamma \bar{z}\,dz$. Which of the integrals is independent of y?

(iii) Use (ii) to show that the conclusion of Cauchy's theorem does not hold for $f(z) = \bar{z}$.

Exercise 2

(i) Calculate $\int_\gamma f(z)\,dz$, where $f(z) = \frac{1}{z}$ and $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$, is the unit circle.

(ii) Use (i) to show that $f(z)$ does not have an antiderivative in its domain of definition.

(iii) Does $f(z) = \frac{1}{z^n}$ have an antiderivative, where $n \ge 2$ is an integer?

(iv) Give an example of a domain Ω, where the function $f(z) = \frac{1}{z(z-1)}$ does not have an antiderivative.

Justify your answer.

Exercise 3

Calculate the residues:

(i) $\mathrm{Res}_0 \frac{\sin(z^3)-e^z}{z^5+z}$;

(ii) $\mathrm{Res}_1 \frac{\cos(2\pi z^2)+z}{e^z-e}$.

Exercise 4

Evaluate the integrals:

(i) $\int_{|z|=2} \frac{e^z}{(z^2-z)(z+3)}\,dz$;

(ii) $\int_0^{2\pi} \frac{\sin^2\theta}{2-\cos\theta}\,d\theta$;

(iii) $\int_{-\infty}^{\infty} \frac{x-x^2}{x^4-2x^2+5}\,dx$.