November 24, 2010 Lecturer Dmitri Zaitsev Michaelmas Term 2010

Course 2325 2010 Complex Analysis I

Sheet 3

Due: at the end of the lecture on Monday in two weeks

Exercise 1

(i) Show that $(e^z)' = e^z$. (Hint. Differentiate in the direction of the x-axis.)

(ii) Let f be any branch of $\log z$ (defined in an open set). Using the fact that f is inverse to e^z, show that f is holomorphic and $f'(z) = \frac{1}{z}$.

Exercise 2

Let γ be the sum of two line segments connecting -1 with iy and iy with 1, where y is a fixed parameter.

(i) Write an explicit parametrization for γ;

(ii) For every y, evaluate the integrals $\int_\gamma z\,dz$ and $\int_\gamma \bar{z}\,dz$. Which of the integrals is independent of y?

(iii) Use (ii) to show that the conclusion of Cauchy's theorem does not hold for $f(z) = \bar{z}$.

Exercise 3

(i) Calculate $\int_\gamma f(z)\,dz$, where $f(z) = \frac{1}{z}$ and $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$, is the unit circle.

(ii) Use (i) to show that $f(z)$ does not have an antiderivative in its domain of definition.

(iii) Use Exercize 1 (ii) to give an example of a domain Ω, where f does have an antiderivative.

(iv) Does $f(z) = \frac{1}{z^n}$ have an antiderivative, where $n \ge 2$ is an integer?

Justify your answer.