Maths 1111/1212: Linear Algebra

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Topics

From the list below, approximately first 8 questions will be covered in Michaelmas term (MA1111), and the remaining part — in the Hilary term (MA1212).

1. Linear algebra in 2d and 3d. Vectors. Dot and cross products.
2. Systems of simultaneous linear equations. Gauss–Jordan elimination.
3. (Reduced) row echelon form for a rectangular matrix. Principal and free variables. Matrix product and row operations. Computing the inverse matrix using row operations.
4. Permutations. Odd and even permutations. Determinants. Row and column operations on determinants. $\det(A^T) = \det(A)$.
5. Minors. Cofactors. Adjoint matrix. Computing the inverse matrix using determinants. Cramer's rule for systems with the same number of equations and unknowns.
6. Fredholm's alternative. An application: the discrete Dirichlet's problem.
7. Coordinate vector space. Linear independence and completeness.
8. Fields: rationals, reals, and complex. Abstract vector spaces. Linear independence and completeness in abstract vector spaces. Bases and dimensions. Subspaces.
9. Linear operators. Matrix of a linear operator relative to given bases. Change of basis. Transition matrices. Similar matrices define the same linear operator in different bases.
10. Kernels and images. Ranks. Dimension formulas.
11. Characteristic polynomials. Eigenvalues and eigenvectors. Diagonalisation in the case when all eigenvalues are distinct.
12. Cayley–Hamilton theorem. Minimal polynomial of a linear operator. Examples (operators with $A^2 = A$).
13. Invariant subspaces. An application: two commuting linear operators have a common eigenvector. Direct sums.
14. Normal form of a nilpotent operator. Jordan normal form (Jordan Decomposition Theorem). Applications: closed expressions for Fibonacci numbers and other recursively defined sequences.
15. Orthonormal bases; Gram–Schmidt orthogonalisation. Orthogonal complements and orthogonal direct sums. Bessel's inequality.
16. Bilinear and quadratic forms. Sylvester's criterion. The law of inertia. Spectral Theorem for symmetric operators.

Textbooks

There will be no lecture notes for this course, so you are encouraged to take notes during the lectures — it takes effort but is really helpful. There are many books which you might find helpful, though they do not correspond exactly to the course content and the order of presentation of topics. For the first part of the course (linear algebra in 2d and 3d, systems of linear equations, operations with matrices, matrices as linear transformations), have a glance at

Anton and Rorres, *Elementary Linear Algebra (applications version).*

For the second part of the course (abstract vector spaces, linear operators, quadratic forms etc.) the exposition will be mostly close to the one from

I.M.Gelfand, *Lectures on Linear Algebra*

(there should be several copies in the College Library, and some 20 copies belonging to the School of Maths are in my office, and you may borrow them as well). As a supplementary material, have a look at

P.Halmos, *Linear Algebra Problem Book*

(some of these problems are quite difficult, but many of them are both approachable and useful).

Lectures and Tutorials

This course takes three hours every week, 1-2 and 5-6 o'clock on Mondays in the Maxwell Theatre, and 5-6 o'clock on Tuesdays in the Large Lecture Theatre of the Chemistry Building (unless otherwise announced; check your College e-mail where I shall send information of any changes that occur). Note that Monday October 26th is a bank holiday, so there will be no lectures then.

The Tuesday linear algebra hour will be a tutorial every other week. For a tutorial, you will get a set of problems close to those from the current homework assignment, and you are supposed to spend time in class solving these problems and discussing solutions with the linear algebra tutors (myself and two postgraduate students). Tutorials are crucial for better understanding of the material explained in class, and they are compulsory, even though not assessed.

Homeworks

Homework assignments will be handed out in class every week. Besides just obtaining answers to questions, you are supposed to justify your answers (in particular, every "yes/no" question also assumes the "why" question). Homeworks are due to hand in after Tuesday's classes; on the same evening I shall post solutions on the course webpage, so late assignments are not accepted.

Exams

There will be an in-class midterm test (for non-TSM students only) sometime during Hilary Term, and a final exam in the end of the year (2 hours for TSM, 3 hours for non-TSM).

Grading

For TSM students, your final grade is either the grade on the final exam or 80% of final exam plus 20% of home assignments grade, whichever is larger. For non-TSM students, your final grade is either the grade on the final exam or 70% of final exam plus 15% of home assignments grade plus 15% of the midterm test result, whichever is larger.

In short, if we denote by H your grade on homework, by MT your midterm result, and by E your final exam result, your overall grade will be obtained from the formula $\max(0.2 \cdot H + 0.8 \cdot E, E)$ for TSM students, and $\max(0.15 \cdot H + 0.15 \cdot MT + 0.7 \cdot E, E)$ for non-TSM students.

Web page

Homework assignments, selected solutions, various handouts and announcements will be posted on the course web page

`http://www.maths.tcd.ie/ vdots/indexLinearAlgebra.html`