

Maths 1212: Linear Algebra II

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Office hours: Monday 10.00–11.00

We shall have classes on Mondays at 1pm in Maxwell Lecture Theatre, Hamilton Building, Tuesdays at 5pm in Large Lecture Theatre in Chemistry Building, and Thursdays 12 noon in Joly Lecture Theatre (until March 17) and Lecture Theatre 11, Lloyd Building (from March 24). There will be a midterm test on March 8: it will take place in Large Lecture Theatre in Chemistry Building, beginning at 5:15pm and ending at 6:45pm.

1. Kernels and images. Ranks. Dimension formulas.
2. Characteristic polynomials. Eigenvalues and eigenvectors. Diagonalisation in the case when all eigenvalues are distinct.
3. Cayley–Hamilton theorem. Minimal polynomial of a linear operator. Examples (operators with $A^2 = A$).
4. Invariant subspaces. An application: two commuting linear operators have a common eigenvector. Direct sums.
5. Normal form of a nilpotent operator. Jordan normal form (Jordan Decomposition Theorem). Applications: closed expressions for Fibonacci numbers and other recursively defined sequences.
6. Orthonormal bases; Gram–Schmidt orthogonalisation. Orthogonal complements and orthogonal direct sums. Bessel’s inequality.
7. Bilinear and quadratic forms. Sylvester’s criterion. The law of inertia. Spectral Theorem for symmetric operators.

Homeworks

Homework assignments will be handed out in class every week. Besides just obtaining answers to questions, you are supposed to justify your answers (in particular, every “yes/no” question also assumes the “why” question). Homeworks are due to hand in on Mondays after the class; on the same day solutions will be posted on the course webpage, so late assignments cannot be accepted.

Assessment

The final mark is $100\% \times \text{final exam mark}$ or $70\% \times \text{final exam mark} + 15\% \times \text{continuous assessment mark} + 15\%$ of the midterm test mark, whichever is higher.

Reading suggestions

There will be no lecture notes for this course, so you are encouraged to take notes during the lectures — it takes effort but is really helpful. There are many books which you might find helpful, though they do not correspond exactly to the course content and the order of presentation of topics. The main textbook to refresh the first half of the course and, possibly, use for supplemental reading in the beginning of the course is *Elementary Linear Algebra (applications version)* by Anton and Rorres. The most relevant textbook for this half of the course is *Lectures on Linear Algebra* by Gelfand (which is entirely different in style from Anton and Rorres, very concise and theoretical). Some of the topics are covered in Gelfand’s book in a too old-fashioned way, and that’s when you should help yourself to the

handouts I prepared. You are also encouraged to attempt problems from *Linear Algebra Problem Book* by Paul Halmos (but do realise that some of these problems are quite difficult, and often much more difficult than typical exam questions).

Online versions of Gelfand's and Halmos's books are available on Google Books via the links

<http://www.tinyurl.com/gelfand-linalg>

and

<http://www.tinyurl.com/halmos-linalg>

respectively. There are also several hard copies of the Gelfand's book owned by the School of Maths available to borrow from my office.

Web page

Homework assignments, selected solutions, reading suggestions, various handouts and announcements will be posted on the course web page

<http://www.maths.tcd.ie/~vdots/indexLinearAlgebra.html>