MA 1111/1212: Linear Algebra
Tutorial problems, March 24, 2011

1. For the matrix $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$, compute the determinants $\Delta_1, \Delta_2, \Delta_3$, and find a triangular change of basis making the matrix of this bilinear form diagonal.

2. Use the Sylvester's criterion to find all values of the parameter a for which the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2ax_1x_2 + 2x_1x_3 + (2-2a)x_2x_3$ on $\mathbb{R}^3$ is positive definite.

3. Let

$$\varphi(x_1, x_2) = \sin^2(x_1 - x_2) - e^{\frac{1}{2}(x_1^2+x_2^2)+2cx_1x_2}.$$

Furthermore, let A be the symmetric 2×2-matrix with entries $a_{ij} = \frac{\partial^2\varphi}{\partial x_i \partial x_j}(0, 0, 0)$.

(**a**) Write down the matrix A.

(**b**) Determine all values of the parameter c for which the corresponding quadratic form is positive definite.

(**c**) (optional) Does φ have a local minimum at the origin $(0, 0)$ for $c = -3/5$?