MA 1111/1212: Linear Algebra
Homework problems due March 29, 2011

Problem 5 is optional. Other problems are assessed.

1. Use the Sylvester's criterion to find all values of the parameter a for which the quadratic form $(18+a)x_1^2+3x_2^2+ax_3^2+10x_1x_2-(8+2a)x_1x_3-4x_2x_3$ on $\mathbb{R}^3$ is positive definite.

2. For each of the following matrices of bilinear forms, compute the determinants $\Delta_1,\dots,\Delta_n$, and find a triangular change of basis where the matrix of this bilinear form is diagonal:

(**a**) $\begin{pmatrix}2&1&0\\1&2&1\\0&1&2\end{pmatrix}$; (**b**) $\begin{pmatrix}1&2&3\\2&2&3\\3&3&3\end{pmatrix}$; (**c**) $\begin{pmatrix}1&1&1\\1&2&2\\1&2&3\end{pmatrix}$; (**d**) $\begin{pmatrix}3&1&1&1\\1&3&1&1\\1&1&3&1\\1&1&1&3\end{pmatrix}$.

3. A bilinear form f is said to be skew-symmetric if $f(v,w)=-f(w,v)$ for all vectors v and w. Prove that f is skew-symmetric if and only if the corresponding quadratic form $q(v)=f(v,v)$ is equal to 0.

4. Let

$$\varphi(x_1,x_2,x_3)=\sin(x_1-x_2)\sin(x_1-x_3)+\sin^2x_2+c\sin x_3\sin(2x_3)+\sin^3(x_1+x_2+x_3).$$

Furthermore, let A be the symmetric 3×3-matrix with entries $a_{ij}=\frac{\partial^2\varphi}{\partial x_i\partial x_j}(0,0,0)$.

(**a**) Write down the matrix A.

(**b**) Determine all values of the parameter c for which the corresponding quadratic form is positive definite.

5. Does the function φ from the previous question have a local minimum at the origin $(0,0,0)$ for $c=1/5$?