MA 1111/1212: Linear Algebra
Tutorial problems, December 7, 2010

1. Let $V = \mathbb{R}^2$, $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ a basis of V, $A\colon V \to V$ a linear operator whose matrix relative to the basis e_1, e_2 is $M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(**a**) Find the transition matrix M_{ef} from the basis e_1, e_2 to the basis $f_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $f_2 = \begin{pmatrix} 4 \\ -9 \end{pmatrix}$, and compute the matrix M_2 of the operator A relative to the basis f_1, f_2.

(**b**) Compute the matrix M_3 of the operator A relative to the basis of standard unit vectors $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

2. The matrix of a linear operator $\mathcal{A}\colon \mathbb{R}^2 \to \mathbb{R}^2$ is (relative to the basis of standard unit vectors) $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$. Compute its matrix relative to the basis (**a**) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

3. A sequence $b_0, b_1, \ldots$ is defined by $b_0 = 0$, $b_1 = 1$, $b_{n+1} = 2b_n + b_{n-1}$.

(**a**) Show that that $\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_n \\ b_{n+1} \end{pmatrix}$.

(**b**) Find eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ and use them to obtain an explicit formula for b_n.