

MA 1111/1212: Linear Algebra
Homework problems due December 7, 2010

1. Compute the dimension and find a basis for the solution space to the linear system $Ax = 0$, where $A = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 0 & -2 \end{pmatrix}$.

2. A 3×3 -checkboard whose cells are filled in with 9 real numbers is called a magic square if all its row sums are pairwise equal, and equal to all of its column sums. Prove that the set of all magic squares forms a subspace of \mathbb{R}^9 , compute the dimension of this space, and find a basis of this space.

3. (a) For $V = \mathbb{R}^2$, find the transition matrix M_{ef} from the basis $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ to the basis $f_1 = \begin{pmatrix} 13 \\ -12 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and find the coordinates of the vector $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ in \mathbb{R}^2 relative to each of the bases e_1, e_2 and f_1, f_2 .

(b) For $V = \mathbb{R}^3$, find the transition matrix M_{ef} from the basis $e_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $e_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ to the basis $f_1 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $f_3 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, and find the coordinates of the vector $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ in \mathbb{R}^3 relative to each of the bases e_1, e_2, e_3 and f_1, f_2, f_3 .

4. (a) Check that the set of all complex numbers forms a 2-dimensional (real) vector space with a basis $1, i$. Compute the matrix (relative to this basis) of linear operator on this space that maps every complex number z to $(3 - 7i)z$.

(b) Check that the set of all quaternions forms a 4-dimensional (real) vector space with a basis $1, i, j, k$. Compute the matrix (relative to this basis) of linear operator on this space that maps every quaternion w to $(2 + i - j + 3k)w$.

(c) Check that the space of all 2×2 -matrices forms a 4-dimensional vector space with a basis $e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, and $e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. Compute the matrix (relative to this basis) of linear operator on this space that maps every 2×2 -matrix X to $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \cdot X - X \cdot \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$.

5. Let $V = \mathbb{R}^3$, $v = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(a) Show that the mapping from V to V given by

$$w \mapsto v \times w$$

is a linear operator, find the matrix of this operator relative to the basis of standard unit vectors, and compute the determinant of this matrix.

(b) Show that the mapping from V to \mathbb{R}^1 given by

$$w \mapsto v \cdot w$$

is a linear operator, and find the matrix of this operator relative to the bases of standard unit vectors in \mathbb{R}^3 and \mathbb{R}^1 .