MA 1111/1212: Linear Algebra
Homework problems due December 13, 2010

Important: this home assignment is due immediately after the 1 o'clock lecture on Monday the 13th of December. Late assignments will not be accepted.

1. Let $V = \mathbb{R}^2$, $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

(**a**) Find the transition matrix M_{ef} from the basis e_1, e_2 to the basis $f_1 = \begin{pmatrix} 13 \\ -12 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(**b**) Find the matrix of the operator A relative to the basis f_1, f_2, if the matrix of A relative to the basis e_1, e_2 is $\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$.

(**c**) Find the matrix of the operator A from the previous question relative to the basis of standard unit vectors $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

2. The (counterclockwise) rotation through $90°$ around the origin is a linear operator from $\mathbb{R}^2$ to $\mathbb{R}^2$. Write down its matrix relative to the basis (**a**) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

3. The matrix of a linear operator $\mathcal{A}\colon \mathbb{R}^3 \to \mathbb{R}^3$ is (relative to the basis of standard unit vectors) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Compute its matrix in the basis (**a**) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

4. A sequence $b_0, b_1, \ldots$ is defined by $b_0 = 0$, $b_1 = 1$, $b_{n+1} = 3b_n - b_{n-1}$.

(**a**) Show that that $\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_n \\ b_{n+1} \end{pmatrix}$.

(**b**) Find eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$ and use them to obtain an explicit formula for b_n.