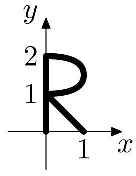
MA 1111/1212: Linear Algebra
Homework problems due October 5, 2010

The last problem is optional. Other problems are assessed.

1. Given that the points $(1,1)$, $(-1,-1)$, and $(0,2)$ are three vertices of a parallelogram, find possible positions of its fourth vertex.

2. Draw the image of the letter R from the picture under the transformation $(x,y) \mapsto (x+2, y+3)$ (all first coordinates of points are increased by 2, all second coordinates — by 3); (**a**) $\ldots (x,y) \mapsto (-x,y)$; (**b**) $\ldots (x,y) \mapsto (x, 2-y)$; (**c**) $\ldots (x,y) \mapsto (y,x)$; (**d**) $\ldots (x,y) \mapsto (2x,2y)$; (**e**) $\ldots (x,y) \mapsto (x,2y)$; (**f**) $\ldots (x,y) \mapsto (x, y+x)$. (**g**) What transforsmation one should apply to get the (Russian) letter Я (on the same place)?



3. Compute the angle between the straight lines connecting the origin $(0,0)$ to the points $(1,1)$ and $(3,5)$.

Consider the vectors $\mathbf{a} = (3,5)$, $\mathbf{b} = (2,3)$, $\mathbf{c} = \mathbf{a} + \mathbf{b}$, $\mathbf{u} = (1,1,2)$, $\mathbf{v} = (2,5,5)$, $\mathbf{w} = (3,2,1)$, $\mathbf{n} = (1,0,1)$. Assume that all the vectors are positioned in such a way that their initial points coincide with the origin.

4. Compute those of the following products which are defined: $\mathbf{a} \cdot \mathbf{c}$, $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \cdot \mathbf{u}$, $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{a} \times \mathbf{b}$, $\mathbf{a} \times \mathbf{w}$, $\mathbf{v} \times \mathbf{w}$, $\mathbf{u} \times \mathbf{w}$, $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$.

5. Compute the area of the parallelogram determined by the vectors (**a**) $\mathbf{a}$ and $\mathbf{b}$; (**b**) $\mathbf{a}$ and $\mathbf{c}$; (**c**) $\mathbf{u}$ and $\mathbf{v}$. (**d**) Compute the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

6. Prove that the coordinates of the point (x', y') where the [counterclockwise] rotation through α around $(0,0)$ brings the given point (x,y) are

$$x' = x \cos\alpha - y \sin\alpha,$$
$$y' = x \sin\alpha + y \cos\alpha.$$

(*Hint*: show that for the points $(x,y) = (1,0)$ and $(x,y) = (0,1)$ directly, and then use the fact that the vector (x,y) is equal to the combination $x \cdot (1,0) + y \cdot (0,1)$.)