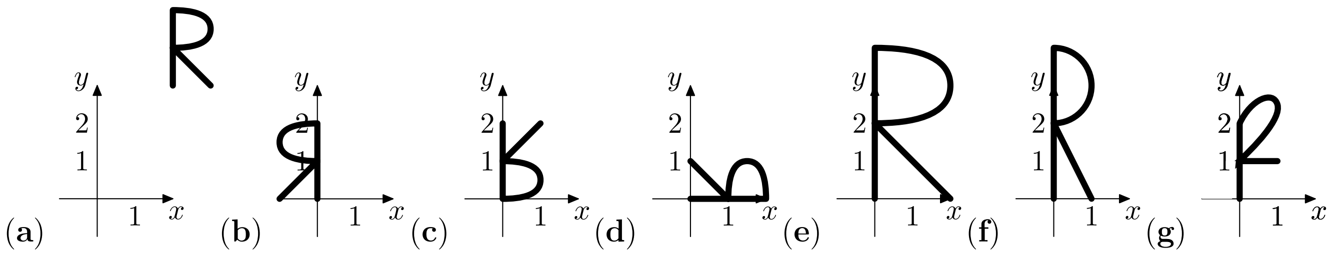
MA 1111/1212: Linear Algebra
Selected answers/solutions to the assignment due October 5

A general remark: for two points $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, the vector with the starting point A and the endpoint B has coordinates $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$.

1. $(-2, 0)$, $(0, -2)$, or $(2, 4)$. In general, if **a**, **b**, and **c** are given points, then the fourth point is one of $\mathbf{a} + \mathbf{b} - \mathbf{c}$, $\mathbf{b} + \mathbf{c} - \mathbf{a}$, and $\mathbf{c} + \mathbf{a} - \mathbf{b}$. One of possible ideas is to use the parallelogram rule carefully. Another idea: the midpoint of the segment connecting (a_1, a_2) to (b_1, b_2) has coordinates $(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2})$; use the fact that the center of a parallelogram is the midpoint of each of its diagonals.

2.



(**h**) the transformation is $(x, y) \mapsto (1 - x, y)$.

3. $(1, 1) \cdot (3, 5) = 8$, $|(1, 1)| = \sqrt{2}$, $|(3, 5)| = \sqrt{3^2 + 5^2} = \sqrt{34}$, so the angle is $\arccos \frac{8}{\sqrt{68}} \approx 0.244$ (in radians).

4. $\mathbf{a} \cdot \mathbf{c} = 55$, $\mathbf{a} \cdot \mathbf{b} = 21$, $\mathbf{a} \cdot \mathbf{u}$ is not defined, $\mathbf{u} \cdot \mathbf{v} = 17$, $\mathbf{v} \cdot \mathbf{w} = 21$, $\mathbf{a} \times \mathbf{b}$ is not defined, $\mathbf{a} \times \mathbf{w}$ is not defined, $\mathbf{v} \times \mathbf{w} = (-5, 13, -11)$, $\mathbf{u} \times \mathbf{w} = (-3, 5, -1)$, $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$ is not defined.

5. (**a**) 1; (**b**) 1; (**c**) $|\mathbf{u} \times \mathbf{v}| = \sqrt{35}$; (**d**) $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}| = 14$.