R \& \text{BACKGROUND}

- For each observable in the $AdS_3$ bulk space, there is, on its conformal boundary, a corresponding observable in its dual $CFT_2$.

$AdS_3 \quad \overset{\text{time}}{\longrightarrow} \quad CFT_2$

- The $CFT_2$ side of the $AdS_3/CFT_2$ duality is not well-known. Some problems include undesirable non-holomorphicity properties and an inability to quantise the theory by any known method. Thus our focus is on the $AdS_3$ bulk space.

- Our goal is to find a superalgebra that is an exact symmetry of the scattering $S$-matrix in the massless $AdS_3$ sector. For this we work in the Hopf algebra framework.

- An advantage of working with Hopf algebras is that it provides a tensor product of representations of a generic (super)algebra in a well-defined way.

- Finding the complete scattering spectrum in $AdS_3$ might reveal some symmetry properties in the dual $CFT_2$.

**HOPF ALGEBRAS**

In this framework, scattering is described by a universal $R$-matrix, which is related to the $S$-matrix by

$S : H_1 \otimes H_2 \rightarrow H_2 \otimes H_1,$

$R : H_1 \otimes H_2 \rightarrow H_1 \otimes H_2,$

where $H_1$ and $H_2$ are representations of the following $2 \rightarrow 2$ scattering procedure:

For a generic Hopf superalgebra $\mathcal{H}$, multiplication has a dual, so-called comultiplication $\Delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H}$. The comultiplications are given by

$\Delta(\mathcal{H}_L) := \mathcal{H}_L \otimes e^{i\xi} + e^{-i\xi} \otimes \mathcal{H}_L,$

$\Delta(\mathcal{H}_R) := \mathcal{H}_R \otimes e^{i\xi} + e^{-i\xi} \otimes \mathcal{H}_R,$

$\Delta(\mathcal{H}) := \mathcal{H} \otimes e^{i\xi} + e^{-i\xi} \otimes \mathcal{H},$

$\Delta(\mathcal{H}^c) := \mathcal{H}^c \otimes e^{i\xi} + e^{-i\xi} \otimes \mathcal{H}^c.$

The above defined algebra is an exact symmetry of the massless $AdS_3$ $S$-matrix, i.e. the comultiplications satisfy the quasicocommutativity relation together with the $R$-matrix, equivalent to the $S$-matrix.

**THE MASSLESS SECTOR**

The massless relativistic Poincaré dispersion relation $E(p) = |p|$ can be related to the massless $AdS_3$ sector via the deformation parameter $q = e^{i\hbar}$, with coupling constant $\hbar$. By $q$-deforming a $(1 + 1)$-dimensional Poincaré superalgebra associated to $sl(1|1)$, non-relativistic regions emerge where the obtained dispersion relation $E(p) = 2\hbar \sin \frac{p}{2}$ is that of the massless $AdS_3$ sector. Taking two copies of this deformed algebra, one identified with an index $L$ (left) and one with an index $R$ (right), the defining anticommutation relations read

$\{\mathcal{H}_L, \mathcal{H}_L\} = 2\hbar,$

$\{\mathcal{H}_R, \mathcal{H}_R\} = 2\hbar,$

$\{\mathcal{H}_L, \mathcal{H}_R\} = 0,$

$\{\mathcal{H}^c_L, \mathcal{H}^c_R\} = 2\hbar.$

**CONCLUSION**

- Starting from a purely relativistic dispersion relation, a certain $q$-deformation allowed for two distinct non-relativistic regions to appear. We approached the scattering spectrum in these regions, where scattering is more intuitive.

- Two copies of the $q$-deformed $(1 + 1)$-dimensional Poincaré superalgebra proved to be an exact symmetry for scattering in the massless $AdS_3$ sector. This was realised in the Hopf algebra framework via the quasicocommutativity relation.

**FUTURE DIRECTIONS**

- Thoroughly analyse the implications of this symmetry algebra on the $CFT_2$ side.

- Find a symmetry algebra for the massless sector of the $AdS_3/CFT_1$ correspondence.

**REFERENCE**


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