Homework 1

Due: October 1st 14:00.
Tutorial: Monday, Sep 30th 17:00 Synge Lecture Hall.
Reading: Jackson sections 1.1-1.5, 1.7.
Reading for next week: Jackson sections 1.8-1.10, 2.1.

Policy

Collaboration is allowed but each student must submit their own version of the solutions.
Extensions of up to a week will be granted upon request before the due deadline and for sufficient cause.
Please include an estimate of the time taken to complete the set and remember to put your name on the solutions.

Problem 1

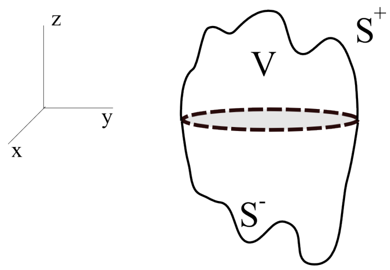
Consider a continuously differentiable vector field, $\vec{A}(\vec{x})$, defined in a closed, bounded region $V \subset \mathbb{R}^3$ whose boundary is a smooth surface S. For each point $P(\vec{x}) \in S$ let $\vec{n}(\vec{x})$ be an outward pointing normal. Then with $d^3x = dxdydz$ being the three-dimensional volume element and da surface area element of S we have the divergence theorem

$$\int_V \vec{\nabla} \cdot \vec{A}(\vec{x})\, d^3x = \int_S \vec{A}(\vec{x}) \cdot \vec{n}\, da \tag{1}$$

Prove this theorem for the case where the vector field only has a z-component i.e.

$$\vec{A}(\vec{x}) = (0, 0, A_z(\vec{x})) \tag{2}$$

and the surface S can be split



into an upper surface S^+ described by the equation

$$\phi(x, y, z) = z - f^+(x, y) = 0 \tag{3}$$

and the a lower surface S^-

$$\psi(x, y, z) = z - f^-(x, y) = 0\ . \tag{4}$$

Extra: Review the proof of Stoke’s theorem.

Problem 2

Consider the time-averaged potential of a neutral hydrogen atom

$$\Phi = \frac{|q|}{4\pi\epsilon_0}\frac{exp(-2r/a_0)}{r}\left(1+\frac{r}{a_0}\right)\ , \tag{5}$$

where q is the charge of the electron (usually denoted e) and a_0 is the Bohr radius. Find the charge distribution (both continuous and discrete parts) that gives rise to this potential. Calculate the total charge of the system.