Interpolation of gridded data using Geostrophic Balance

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Abstract:

In this report, a method of interpolating data from the grid of a global climate model (GCM) to that of a regional climate model (RCM) is outlined. This method forces the wind fields into geostrophic balance, so that the actual balance between the pressure field and the wind fields in the atmosphere is respected. This geostrophic forcing is thus a means of intializing the data so that the RCM can be integrated forward in time. Initialization is important in eliminating spurious gravity waves, and geostrophic balance could in fact be an efficient way of providing this initialization.

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1. Introduction

When modelling the atmosphere using the primitive Shallow Water Equations (SWEs), we are faced with the challenge of refining the initial conditions so that they do not support gravity waves. For, the high-frequency gravity wave solutions are not the determinant of atmospheric conditions; these being determined by low-freqency or Rossby waves.

The initial conditions for the RCM are provided by a GCM. The fields defined at the global gridpoints are interpolated to the RCM gridpoints. At present, data-assimilation cycles ensure that the initial conditions do not produce noisy foreward integrations (i.e. those supporting gravity waves). A more efficient (less time-consuming) means of initialization might be to force geostrophic balance.

The geostrophic interpolation scheme is explained below and some results are presented. It would be desirable to compare these results with current interpolation schemes; progress towars this aim is outlined too.

2. The Model Grid and the Momentum Equation

Models such as the HIRLAM model use as coordinates latitude, longitude and eta. For, any monotone function of the height, z, can be used as a vertical coordinate, and eta is such a function. It is defined by the relation

$$p(\lambda,\phi,z)=A(\eta)+B(\eta)\,p_s(\lambda,\phi) \qquad (1)$$

Where p_s is the surface pressure at latitude ϕ and longitude λ. Surfaces of constant eta follow the orography: at small heights they hug the surface and in the upper reaches of the atmosphere they are planar. The functions A and B are defined such that at the surface, A = 0 and B = 1 and at the top of the model domain, A = 1 and B = 0.

It is convenient to introdue a rotating Cartesian frame, fixed to the earth's surface. With respect to this frame, the elements of length dx and dy are given by the following formulas:

$$dx=ad\phi \quad dy=acos\phi\, d\lambda \qquad (2)$$

Where a is a suitable value of the earth's radius.

Relative to this frame, the Euler momentum equation (together with the Coriolis force) in the horizontal direction is

$$\frac{d\boldsymbol{v}}{dt}=-f\,\boldsymbol{k}\times\boldsymbol{v}-\nabla_{\eta}\Phi-R_d T_V\nabla_{\eta}\ln p \qquad (3)$$

Where R_d is the gas constant for dry air, $T_V=T+0.61q$ is the virtual temperature, q is the specific humidity, $f=2\Omega\sin\phi$ is the Coriolis parameter and the gradient ∇_η is at constant eta. The vector $\boldsymbol{v}$ is a two-vector: $\boldsymbol{v}=(u,v)$. The geopotential is $\Phi=gz$.

If the pressure gradient and the Coriolis forces balance, then the total time derivative $\frac{d\boldsymbol{v}}{dt}$ vanishes. This provides a diagnostic relation for the wind fields (geostrophic balance):

$$\begin{aligned} u&=-\frac{1}{f}\left(\frac{\partial\Phi}{\partial y}\right)_\eta-\frac{R_d T_V}{fp}\left(\frac{\partial p}{\partial y}\right)_\eta \\ v&=+\frac{1}{f}\left(\frac{\partial\Phi}{\partial x}\right)_\eta+\frac{R_d T_V}{fp}\left(\frac{\partial p}{\partial x}\right)_\eta \end{aligned} \tag{4}$$

The geopotential Φ at any eta level can be computed by hydrostatic balance. For, in the vertical direction, the pressure difference across an infinitesimally thin column of air is equal to the gravitational force on that column, so that the atmosphere is in hydrostatic balance. In symbols,

$$\begin{aligned} &\delta p=-\rho g\delta z \\ &\Rightarrow\delta p=-\rho\delta\Phi \end{aligned} \tag{5}$$

Thus, on integration,

$$\Phi=\Phi_s-\int_{p_s}^{p(\eta)}\frac{dp}{\rho}=\Phi_s-\int_{p_s}^{p(\eta)}R_d T_V\frac{dp}{p}=\Phi_s-\int_{p_s}^{p(\eta)}R_d(T+0.61q)\frac{dp}{p} \tag{6}$$

Indexing the *N* eta levels like $\eta_{0,}\eta_{1,}\cdots,\eta_{N-1}$, with η_0 at the surface and η_{N-1} in the upper reaches of the atmosphere, we may replace equation (6) by a finite sum, incurring an error $\epsilon\sim max_{i=0,1,2,\cdots N-1}|p_{i+1}-p_i|$.

$$\Phi_n=\Phi_s-\sum_{i=0}^{n}R_d(T_i+0.61q_i)\left(\frac{p_{i+1/2}-p_{i-1/2}}{p_{i+1/2}+p_{i-1/2}}\right) \tag{7}$$

Equation (7) gives the geopotential on the *n*th eta level in terms of the pressure on the half-levels. The pressure is known on the half-levels because the *A's* and *B*'s are defined for the half levels in models like the HIRLAM model:

$$p_{i+1/2}=A_{i+1/2}+B_{i+1/2}\,p_s(\lambda,\phi) \tag{8}$$

The pressure on the full levels is then obtained by a simple average:

$$p_i = \frac{1}{2}\left(p_{i+1/2} + p_{i-1/2}\right) \qquad i = 1,2,\cdots,N-1 \tag{9}$$

3. The Interpolation

We now have sufficient information to carry out the interpolation. We imagine a Cartesian grid, where each grid point is determined uniquely by a latitude and longitude. The domain of the RCM is contained within the domain the domain of the GCM. We carry out the interpolation in the following steps:

1. Interpolate the surface fields p_s and Φ_s to the RCM grid (RCMG) using bilinear interpolation.
2. Get the values of pressure on the RCMG and the GCM grid (GCMG) using equations (1) and (9), now that the surface pressure on the RCMG is known.
3. Get the value of the geopotential gradients and pressure gradients on the GCMG.
4. Get the values of geopotential on the GCMG using the finite sum in equation (7).
5. Define a grid with the spatial resolution of the GCM and with the eta levels of the RCM. Interpolate temperature (*T*), specific humidity (*q*) and each of the gradients from the GCMG to this new grid in the vertical direction only. We use the value of pressure on the eta levels as a coordinate because this is known exactly.
6. Now interpolate the fields *T*, *q*, $\left(\frac{\partial \Phi}{\partial x}\right)_\eta$, $\left(\frac{\partial \Phi}{\partial y}\right)_\eta$, $\left(\frac{\partial p}{\partial x}\right)_\eta$, $\left(\frac{\partial p}{\partial y}\right)_\eta$, from the new grid to the RCMG in the horizontal direction.
7. *T* and *q* are now known on the RCMG and so the geopotential on said grid can now be obtained.
8. Using the fields in (6), now defined on the GCMG, we can compute the geostrophic wind components *u* and *v* from the equations in section 2.

Note the order of the interpolation: vertical first, then horizontal.

The gradients of pressure and geopotential are computed using centred differences for the interior of the GCM domain and using left / right hand differences on the boundary. In general, if (x_i, y_i) is a grid point described in terms of our Cartesian system, (x_i, y_{i+1}) and (x_{i+1}, y_i) are adjacent grid points, and $\psi(x, y, \eta)$ is any scalar field, then we have the following gradients:

$$\left(\frac{\partial \psi}{\partial x}\right)_\eta = \frac{\psi(x_{i+1}, y_i) - \psi(x_{i-1}, y_i)}{2\Delta x} + o(\Delta x^2), \quad \Delta x = a\Delta\phi \qquad \text{in the domain,}$$

$$\left(\frac{\partial \psi}{\partial x}\right)_\eta = \frac{\psi(x_{i+1}, y_i) - \psi(x_i, y_i)}{\Delta x} + o(\Delta x) \qquad \text{for } (x_i, y_i) \text{ on the boundary.}$$

$$\left(\frac{\partial \psi}{\partial y}\right)_\eta = \frac{\psi(x_i, y_{i+1}) - \psi(x_i, y_{i-1})}{2\Delta y} + o(\Delta y^2), \quad \Delta y = a\cos\phi\,\Delta\lambda$$ inside the domain

&c.

(10)

Bilinear interpolation is used for the interpolation in the horizontal direction. Suppose that a continuous function *f*(*x*,*y*) is known at the adjacent grid points $(x_{1,} y_1)$, $(x_{2,} y_2)$, $(x_{3,} y_3)$, $(x_{4,} y_4)$ so that $f_i \equiv f(x_i, y_i)$ for *i* = 1, 2, 3, 4. The significance of the labelling is shown in figure 1. If the point (*x*,*y*) lies in the grid square, the value *f*(*x*,*y*) of the function at this point is got as follows:

$$t = \frac{x - x_1}{x_2 - x_1}$$
$$u = \frac{y - y_1}{y_2 - y_1} \quad (11)$$

$$f(x, y) = (1-t)(1-u)\,f(x_{1,} y_1) + t(1-u)\,f(x_{2,} y_2) + tuf(x_{3,} y_3) + (1-t)uf(x_{4,} y_4)$$

(12)

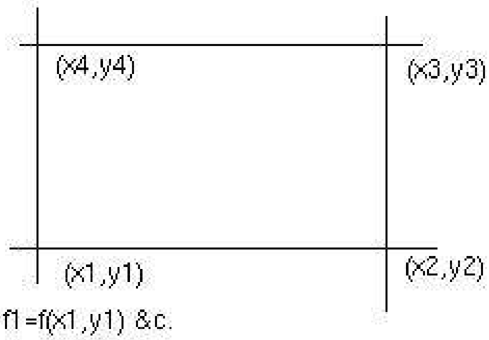

Figure 1: The grid square. The labelling of the grid points corresponds to the indices in equations (11) and (12).

This method has one disadvantage however: if the field *f* is C^1 then the interpolated field is only C^0 [1]: in any grid square the derivatives of the field exist. However, they vary discontinuously as one wanders from grid square to grid square. Thus, in our interpolation scheme, we compute the GCMG field gradients first, and then interpolate these gradients to the RCMG, rather than interpolating fields and then computing gradients. In reality, we have a commutative diagram (figure 2); the non-commutativity implied in this discussion is merely an artifice of the bilinear interpolation.

$$\begin{array}{ccc}
\Phi^{GCM}, p^{GCM} & \overset{interpolation}{\rightarrow} & \Phi^{RCM}, p^{RCM} \\
\frac{\partial}{\partial x^i}\downarrow & & \downarrow\frac{\partial}{\partial x^i} \\
\frac{\partial \Phi^{GCM}}{\partial x^i}, \frac{\partial p^{GCM}}{\partial x^i} & \overset{interpolation}{\rightarrow} & \frac{\partial \Phi^{GCM}}{\partial x^i}, \frac{\partial p^{GCM}}{\partial x^i}
\end{array}$$

Figure 2: The commutative diagram for the interpolation.

The piecewise linear interpolation in the vertical direction is the one-dimensional analogue of the bilinear interpolation. Given a continuous function *g*(*x*) defined at two distinct points such that $g(x_1)=g_{1,}\ g(x_2)=g_2$ and a point $x\in[x_{1,}x_2]$, we compute *g*(*x*) in the following manner:

$$\begin{gathered}
g(x)=g(x_1)+g'(x_1)(x-x_1)+o(|x-x_1|) \\
g'(x_1)\approx\frac{g(x_2)-g(x_1)}{x_2-x_1}
\end{gathered} \tag{13}$$

Again, this method fails to preserve the C^1 -ness of C^1 functions, but this does not present a problem for us here.

4. Code:

The interpolation scheme is written in C. The source and a readme document can be obtained at www.maths.tcd.ie/~tkachev/Meteorology.html

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1 C^0 : A continuous function. C^1 : A continuous function with continuous first derivative.

5. Results:

The results of the interpolation are presented for input data (GCM) coming from ECMWF analysis valid 1st Jan 1993, 00UTC. The RCM uses the HIRLAM eta levels. The interpolation is performed over the sea (negligible orography). The assumption of geostrophic balance is especially valid here, since the pressure and geopotential gradients over the sea have orders of magnitude which produce a geostrophic wind whose magnitude is of the correct order:

$$\begin{aligned} \|\nabla_\eta p\| &\sim 0.003 \\ \|\nabla_\eta \Phi\| &\sim 0.0003 \\ f &\sim 0.001 \\ \|\boldsymbol{v}\| &\sim 10 \end{aligned} \tag{14}$$

In SI units, at middle latitudes in the Northern Hemisphere, near the earth's surface.

The domain chosen for the RCM is -50 W to -40W, 30 N to 40 N (mid-Atlantic).

Some results are presented in graphical form.

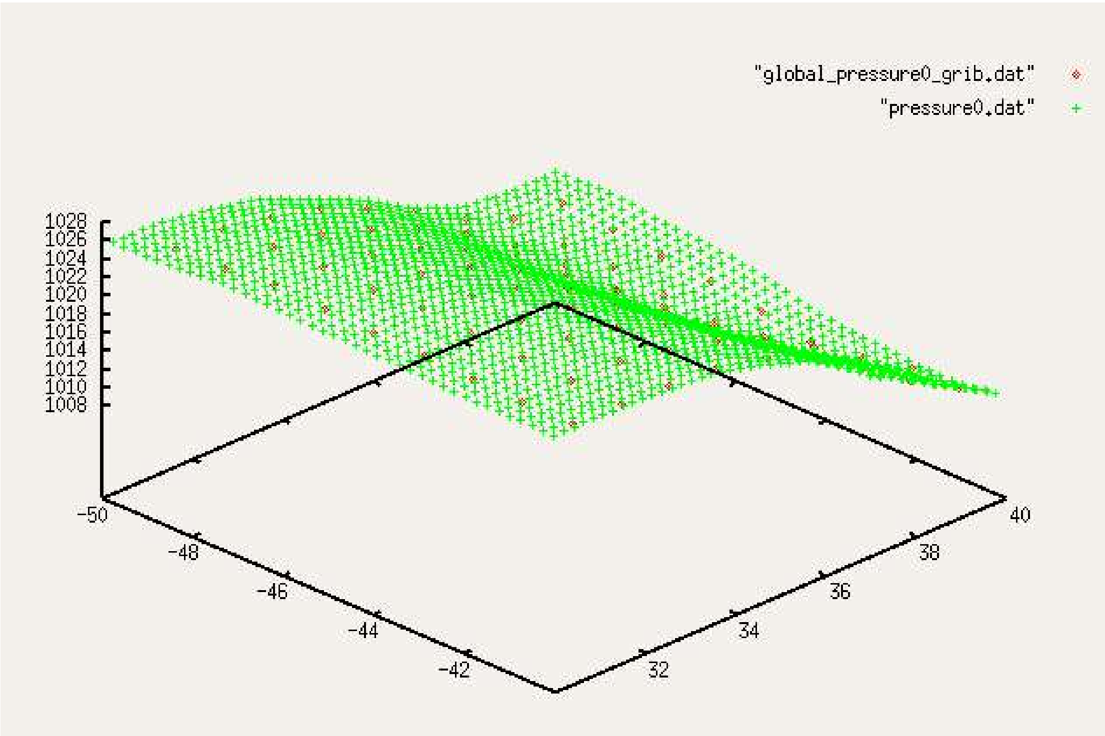

Figure 3: The interpolated field of surface pressure (green) compared with the GCM temperature (red), for the model domain chosen.

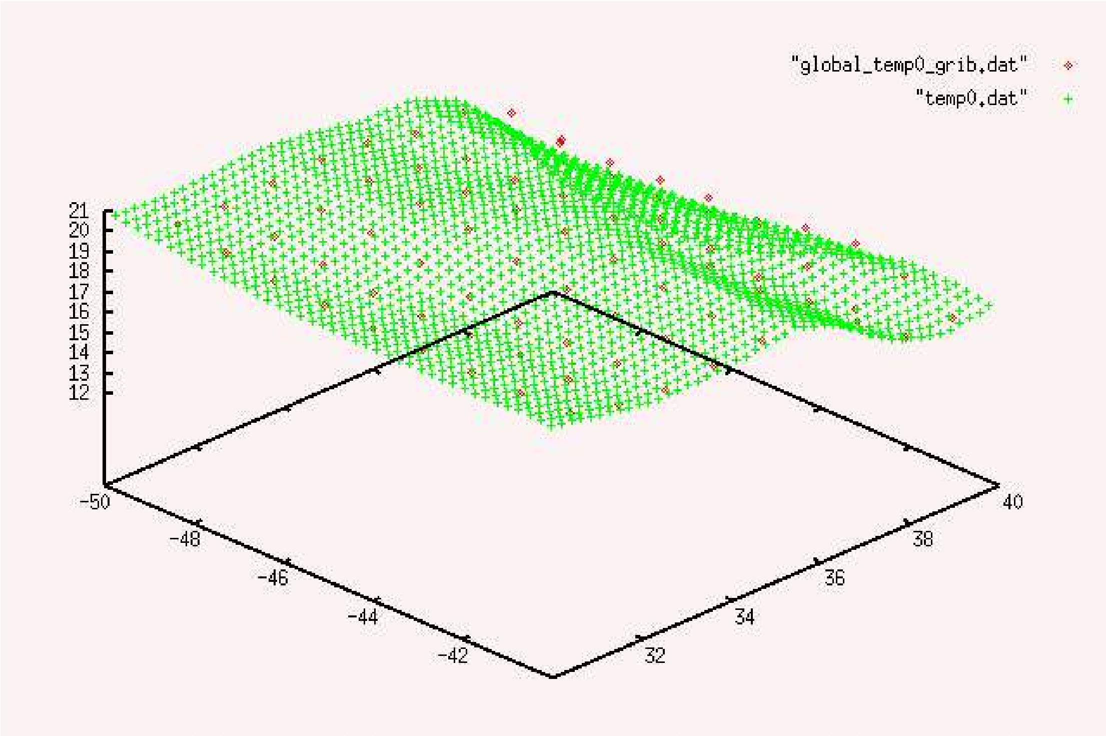

Figure 4: The interpolated field of surface temperature (green) compared with the GCM temperature (red), for the model domain chosen.

The geostrophic wind components were obtained by the method outlined above. However, the wind fields were also interpolated in the conventional way, by first interpolating piecewise linearly in the vertical direction and then by interpolating bilinearly in the horizontal direction. We therefore have two wind fields, which differ by up to 20%. The wind fields to be used as initial conditions in the forward integration of the SWEs are then a weighted average of these two wind fields:

$$\boldsymbol{v_i} = w_i \boldsymbol{v_i^{CH}} + (1 - w_i)\boldsymbol{v_i^{GB}} \tag{15}$$

Where, in an obvious notation, CH refers to the conventional horizontal interpolation and GB refers to the geostrophic wind. The label *i* indexes the eta levels. w_i is a weight, chosen so that on the surface, $\boldsymbol{v_s} = \boldsymbol{v_s^{CH}}$ while at the top of the domain, $\boldsymbol{v_{top}} = \boldsymbol{v_{top}^{GB}}$. Thus, we chose the following weight:

$$w_i = \cos\left(\frac{i\pi}{2(N-1)}\right) \tag{16}$$

The vertical profile of the wind speed $\sqrt{u^2+v^2}$ for the coordinate (-50W,30N) is presented below.

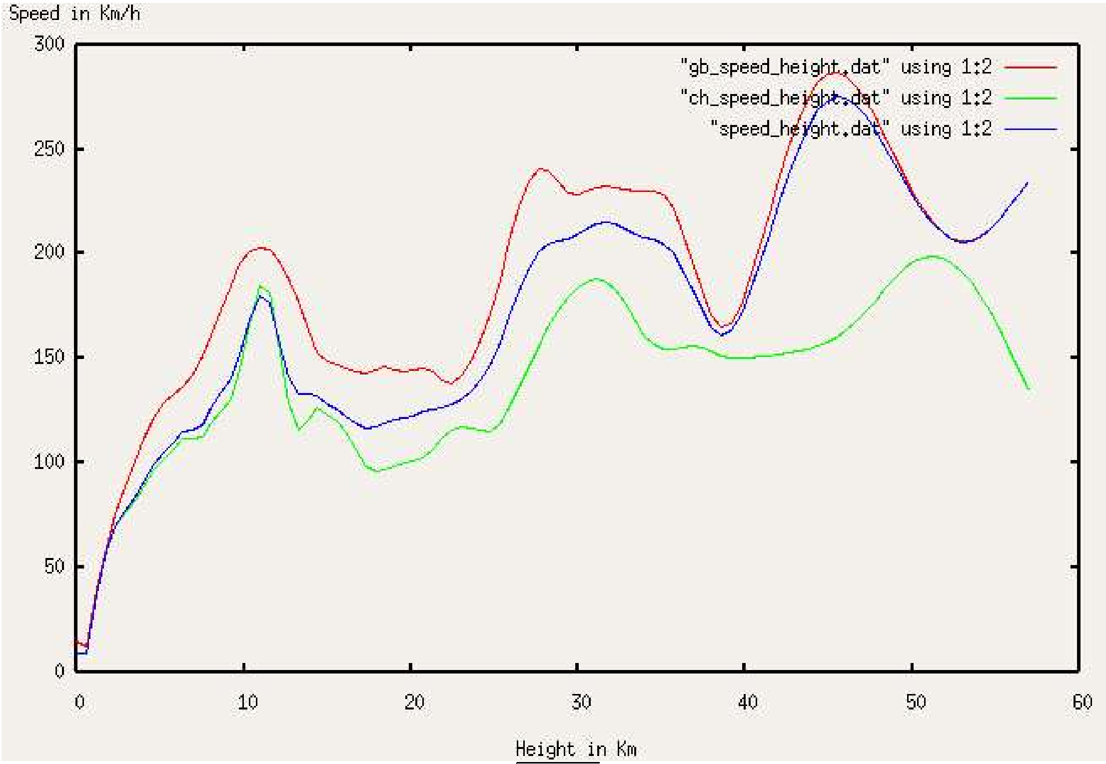

Figure 5: Vertical Profile of speed $\sqrt{u^2+v^2}$ *for grid point (-50W, 30N). The red curve is the geostrophic wind, the green curve is the ageostrophic wind while the blue curve represents the weighted average. Note that the red curve is smooth while the others are not; this is a consequence of the piecewise linear interpolation used in the vertical interpolation.*

6. Further Investigations

Having devised a successful interpolation scheme for the GCM-RCM interpolation, it would be interesting to compare this new scheme with previous method, such as that used in the HIRLAM model. It will be necessary to write C code to call the FORTRAN routines of the HIRLAM model. This program will read input data from the command line and will output data to files whose content will be plotted. This is being attempted at present.

Finally, the forward integrations produced by the new scheme should be compared to those produced by the HIRLAM interpolations. This is a bigger task, although given the three hours needed for data assimilation in HIRLAM operational forecasts, it is a worthy undertaking.

The scheme proposed here has one physical condition in the horisontal, viz. geostophic balance. Further conditions might be added: for example, the C^1 -ness of the fields in the vertical direction could be forced by using a more sophisticated method of vertical interpolation, or by forcing the fields to satisfy continuous lapse-rate conditions, e.g.

$$\begin{aligned} \frac{\partial T}{\partial z} &= -\gamma \\ T &= T_s - \int_{z_{surf}}^{z} \gamma \, dz \end{aligned} \tag{17}$$

7. Conclusion

The geostrophic forcing of the wind fields in the atmosphere was investigated to produce a novel means of interpolating data from a GCMG to a RCMG. The resulting discretized fields were plotted and were found to coincide with the GCM fields. A profile of the geostrophic wind for a certain RCM grid point was compared with the other fields of wind speed, obtained by conventional interpolation and by weighted averaging. Since the trends in this profile agrees, the forcing of geostophic balance is a reasonable assumption. The practical usefulness of this forcing, however, can only be ascertained when forward integrations of the SWEs arising from the new scheme are compared with previous methods of assimilation.