The Hall Effect

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Abstract:

This experiment studies the Hall effect – the production of an **E** field in a conductor due to the presence of both current and a **B** field – in semiconductor materials. Both an electromagnet and a ferromagnetic material will be used as sources of magnetic field. The presence of a residual field inside the electromagnet will be noted, the basic equations of the Hall effect will be verified with respect to the electromagnet and the ferromagnetic material. Finally, the properties of a lock-in amplifier (LIA) will be investigated.

Basic Theory and Equations:

The Hall effect is analogous to the deflection of a moving electron due to the presence of electric and magnetic fields. The electron undergoes deflection determined by the relative strength of the two fields. In a similar manner, the moving charges in a current-carrying conductor, in the presence of an external **B** field undergo Lorentz-force deflection, given by $q\mathbf{v}_d \times \mathbf{B}$, where q is the charge on each particle and $\mathbf{v}_d$ is the drift velocity. As in figure (1), the charges are deflected and, owing to the finite extent of the conducting material, they accumulate as shown.

Figure (1)

Such a distribution of charge induces an **E** field, the Hall field, and the charge accumulation stops when the electric and magnetic forces balance, i.e. when

$$q\mathbf{E} = q\mathbf{v}_d \times \mathbf{B} \qquad (1)$$

Now $\mathbf{J} = Nq\mathbf{v}_d$, where N is the number of charges per unit volume, so the Hall field is given by $E = \frac{1}{Nq} JB$, assuming that **J** and **B** are orthogonal. Now $V = Ew$, where w is the distance through which the field moves a given charge, and $I = Jwt$, where t is the thickness of the material. We therefore have the *Hall voltage*:

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$$V_H = \frac{1}{Nqt} IB \qquad (2)$$

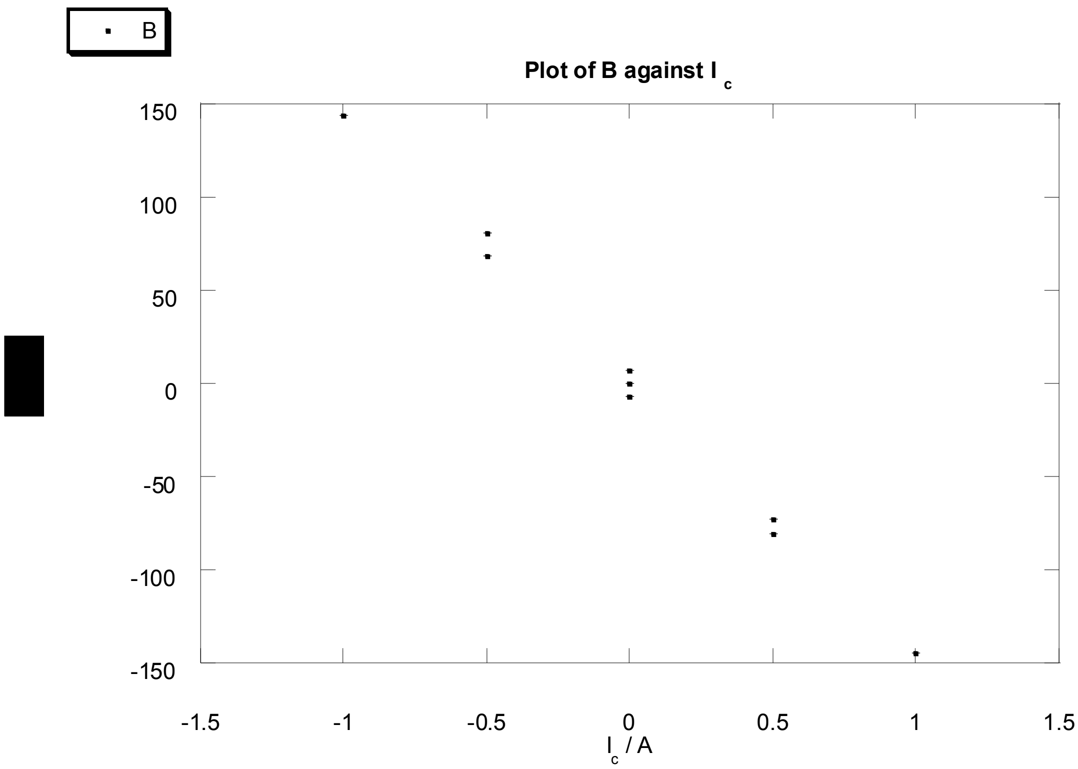
This equation suggests that $V \propto IB$, which we shall verify in the course of the experiment. Note also that equation (2) has q in the denominator ($\frac{1}{Nqt}$) and in the numerator, via $I = wtNqv_d$, and so the induced voltage is independent of the sign of the charge q.

(i) The characteristics of an electromagnet:

The field induced in an electromagnet, due to the current passed through the coils thereof, is investigated.

The current in the coils, I_C, is varied, from between 1A and -1A.

Figure (2):



$\delta B = \pm 0.05 mT$

We see that I_C is varied from zero→+1A→zero→-1A→zero. We see that B as a function of I_C is multi-valued, and that there is no unique value of the residual field (the field present when I_C is zero). Since there is no definite relationship between I_C and B, in the proceeding experiments, B must be measured directly using the gaussmeter.

(ii) The Hall voltage measured with the electromagnet:

The voltage across the sample of germanium is measured. This is V_{34}. Due to a slight misalignment of the terminals 3 and 4, a field-independent voltage V_0 contributes to V_{34}, so that

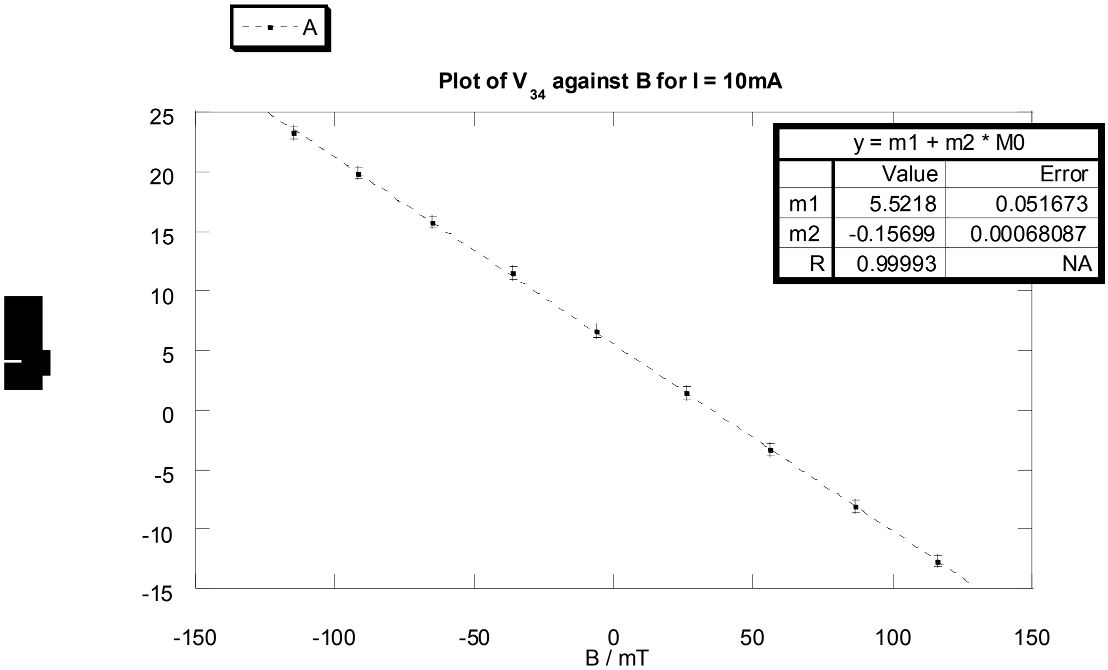
$$V_{34} = V_H + V_0 = \alpha_H BI + \beta I \qquad (3)$$

Where $\alpha_H = \dfrac{1}{Nqt}$ and β are constants of proportionality.

V_{34} is plotted, as a function of B, for three values of I – 10, 20 and 30 mA. The resulting graphs are shown below, from whence we obtain a value of α_H, which is independent of the current I, to within experimental error.

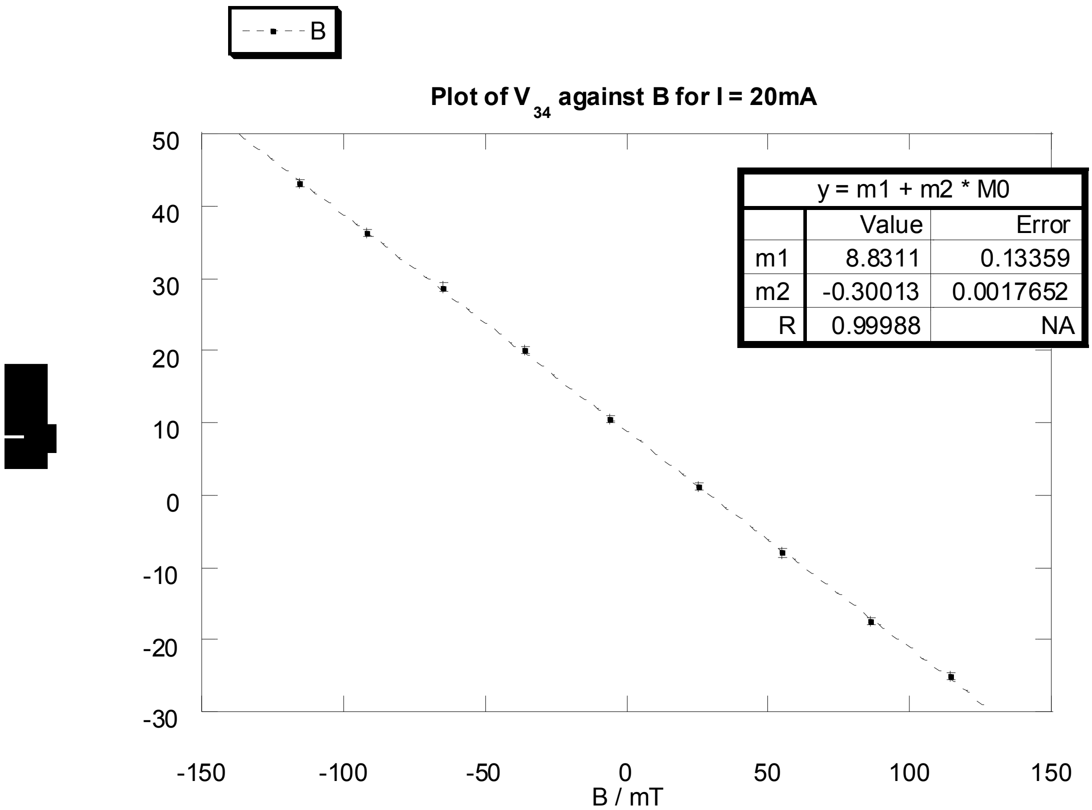
$$m \equiv Slope$$
$$V_0 \equiv V_{34}(B=0)$$

Figure (3.1)



$$I = 10 \pm 0.5mA$$
$$m = -0.15699 \pm 0.00068T/V = \alpha_H I$$
$$\Rightarrow \alpha_H = -15.7 \pm 0.8V/TA$$
$$V_0 = 5.5218 \pm 0.0518mV = \beta I$$
$$\Rightarrow \beta = 0.55 \pm 0.03V/A \equiv 0.55 \pm 0.03\Omega$$

Figure (3.2)



$I = 20 \pm 0.5 mA$

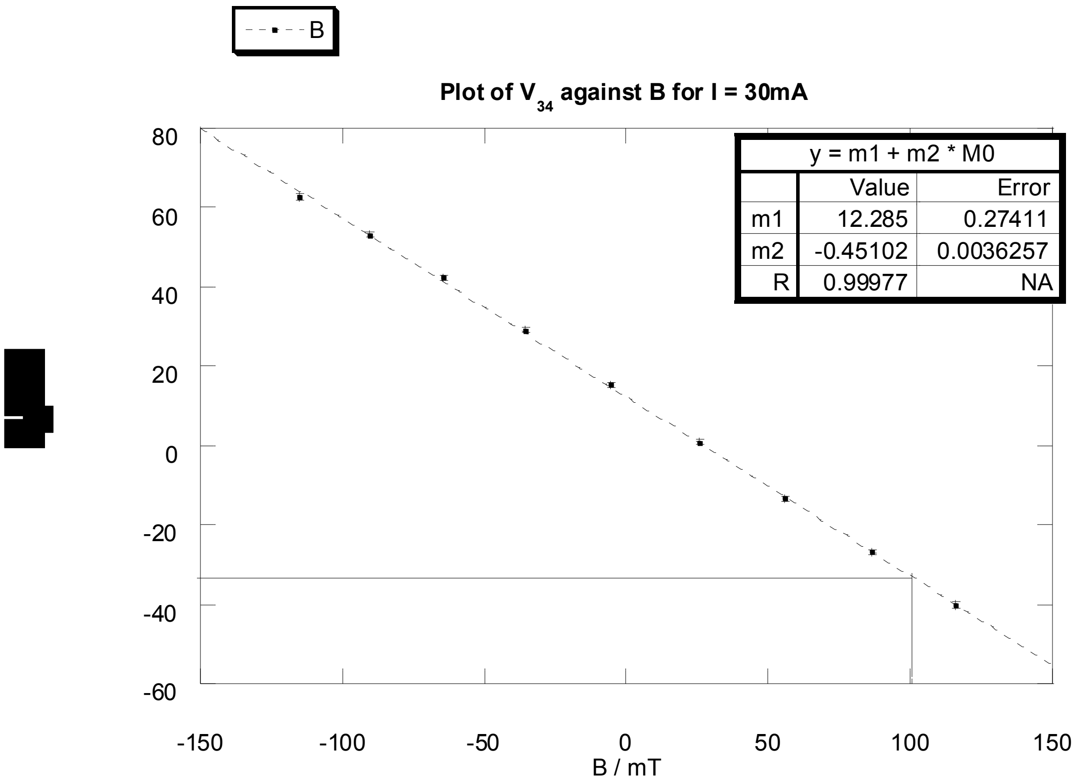
$m = -0.30013 \pm 0.00177 T/V = \alpha_H I$

$\Rightarrow \alpha_H = -15.0 \pm 0.4 V/TA$

$V_0 = 8.8311 \pm 0.13359 mV = \beta I$

$\Rightarrow \beta = 0.44 \pm 0.02 V/A \equiv 0.44 \pm 0.02 \Omega$

Figure (3.3)



$$I = 30 \pm 0.5 mA$$
$$m = -0.45102 \pm 0.00363 T/V = \alpha_H I$$
$$\Rightarrow \alpha_H = -15.0 \pm 0.4 V/TA$$
$$V_0 = 12.285 \pm 0.27411 mV = \beta I$$
$$\Rightarrow \beta = 0.41 \pm 0.02 V/A \equiv 0.41 \pm 0.02 \Omega$$

$$\bar{\alpha}_H = \frac{-15.7 - 15.0 - 15.0}{3} \pm \frac{\sqrt{0.8^2 + 0.4^2 + 0.4^2}}{3} V/TA = -15.3 \pm 0.3 V/TA$$

$|\bar{\alpha}_H| = \frac{1}{N|q|t} \Rightarrow N = \frac{1}{|\bar{\alpha}_H| et}$, on taking $|q| = e = 1.6 \times 10^{-19} C$.

Thus,

$$N = \frac{1}{(15.3 \pm 0.3 V/TA)(1.0 \times 10^{-3} \pm 0.02 \times 10^{-3} m)(1.6 \times 10^{-19} C)} = 4.0 \times 10^{20} \pm 0.2 \times 10^{20} / m^3$$

This value compares with canonical value $1.0 \times 10^{21} / m^3$ (1)

Note that we have found that $\bar{\alpha}_H < 0$. We can clearly see this from the last graph, where for *B = 100mT, $V_{34}<0$.* This demonstrates that the sign of the charge carriers in germanium is negative, i.e. the charge carriers are electrons.

Now the values of β do not agree to within experimental error, and this is unexplained. We therefore average over the two values of β which do agree, to get

$$\bar{\beta} = \frac{0.44+0.41}{2} \pm \frac{0.02}{\sqrt{2}}\Omega = 0.43 \pm 0.01\Omega$$

Now in equation (3), β is the resistance (*R*) of the germanium sample, as measured between terminals 1 and 2. So the resistivity, ρ, is $\frac{RA}{l} = \frac{Rwt}{l}$, which implies the following equation:

$$\sigma \equiv \frac{1}{\rho} = \frac{l}{Rwt} \qquad (4)$$

(σ is measured in Siemans: $1S = 1\ m^{-1}\ \Omega^{-1}$).

Thus,

$$\sigma = \frac{\left(10.0\times10^{-3} \pm 0.02\times10^{-3}\right)}{\left(0.43\pm0.01\right)\left(5.0\times10^{-3} \pm 0.02\times10^{-3}\right)\left(1.0\times10^{-3} \pm 0.02\times10^{-3}\right)} = \left(4.65 \pm 0.25\times10^{3}\right)S$$

Where $\delta\sigma = \frac{w}{Rlt}\left(\frac{\delta w}{w} + \frac{\delta R}{R} + \frac{\delta l}{l} + \frac{\delta t}{t}\right)$

Now the *carrier mobility*, μ, is defined as

$$\mu \equiv \frac{\sigma}{Nq} \qquad (5)$$

Thus, we have the following:

$$\mu = \frac{\left(4.65\pm0.25\times10^{3}\right)\Omega^{-1}m^{-1}}{\left(4.0\times10^{20} \pm 0.2\times10^{20}/m^{3}\right)\left(1.6\times10^{-19}C\right)} = 727 \pm 75\frac{m^2}{\Omega C}$$

(iii) Measurement of the Hall voltage using a "magic cylinder"; using the Lock-In Amplifier (LIA):

This part of the experiment studies the Hall voltage of a crystal of germanium, with respect to a permanent magnet, called a "magic cylinder". This consists of a cylinder of eight component magnets, each with a permanent magnetic field of different orientation. These **B** fields add to give a net **B** field in a fixed direction.

The apparatus is set up as in figure (4). The **B** field is set perpendicular to the sample. This is arranged by making V_{34} maximal.

Figure (4)

We obtain the following results in tabular form:

I / mA	$V_{34}(1)$ / mV	$V_{12}(1)$ / mV	$V_{34}(2)$ / mV	$V_{12}(2)$ / mV	V_H/mV	V_0/mV	V_0/V_{12}
5	21.3	0.55	0.2	0.55	10.6	10.6	19.54
10	42.2	1.06	1.5	1.06	20.25	21.85	20.61
15	64.5	1.59	0.40	1.59	32.05	32.05	20.40
20	88.1	2.12	7.8	2.12	42.99	42.99	19.86
25	112.4	2.67	12.4	2.71	50.0	62.4	18.45

δI / mA	$\delta V_{34}(1)$ / mV	$\delta V_{12}(1)$ / mV	$\delta V_{34}(2)$ / mV	$\delta V_{12}(2)$ / mV	δV_H/V	δV_0/V	$\delta(V_0/V_{12})$
0.1	0.1	0.1	0.1	0.1	0.1	0.1	3.72
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.19
0.1	0.1	0.1	0.1	0.1	0.1	0.1	1.28
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.93
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.19

Where the following equations

$$V_H = \frac{V_{34}(1) - V_{34}(2)}{2}$$

$$V_0 = \frac{V_{34}(1) + V_{34}(2)}{2}$$

follow by taking V_H to the the time-average of the oscillating signal (see figure (5)), and where the 1 and 2 indicate the different polarities (directions of **B**), at which V_{34} is measured.

Figure (5)

We have the fixed value of B, $B = \pm 170mT$.

I / A	V_H / mV	α_H / V/TA	$\delta(\alpha_H)$ / V/TA	V_0 / V	V_0/I / Ω	$\delta(V_0/I)$ / Ω
0.005	10.6	12.47	0.37	10.6	2.12	0.06
0.010	20.25	11.91	0.22	21.85	2.04	0.04
0.015	32.05	12.57	0.31	32.05	2.14	0.02
0.020	42.99	12.64	0.08	42.99	2.15	0.03
0.025	50.0	11.75	0.07	50.0	2.00	0.01

$$\alpha_H = \frac{V_H}{BI}$$

$$\overline{\alpha}_H = \frac{12.47+11.91+12.57+12.64+11.75}{5} \pm \frac{\sqrt{0.37^2+0.22^2+0.31^2+0.08^2+0.07^2}}{5}$$

$$\Rightarrow \overline{\alpha}_H = 12.3 \pm 0.1 V/TA$$

Similarly, $\overline{\beta} \equiv R = 2.1 \pm 0.01\Omega$

We see that the $\alpha_H = \dfrac{1}{Nqt}$ is positive, so we conclude that the charge carriers are positive. Since the material being investigated is a semiconductor, the charge carriers are *holes*.

$$\sigma = \frac{1}{\rho} = \frac{l}{Rwt} = \frac{\left(10.0\times10^{-3} \pm 0.02\times10^{-3}\right)}{(12.3\pm0.1)\left(5.0\times10^{-3} \pm 0.02\times10^{-3}\right)\left(1.0\times10^{-3} \pm 0.02\times10^{-3}\right)} = 162 \pm 5S$$

$$\mu \equiv \frac{\sigma}{Nq} = \sigma t \alpha_H = \left(162 \pm 5\Omega^{-1}\right)\left(1.0\times10^{-3} \pm 0.02\times10^{-3} m\right)\left(12.3 \pm 0.1 V/TA\right) = 2.0 \pm 0.1 m^2/\Omega C$$

The above measurements were repeated, using a digital oscilloscope and the following data were obtained:

I/mA	$V_{34}(1)/mV$	$V_{34}(2)/mV$	$\delta V_{34}/mV$
5	20.8	0	1.0
10	43.2	0	1.0
15	65.66	3.2	2.0
20	90.0	4.0	2.0
25	112.0	10	2.0

We see that these values of voltage agree, to within experimental error, to the values of V_{34} previously obtained. The oscillator must be DC-coupled because V_{34} consists of two components – the time-varying Hall voltage, and the constant voltage V_0, due to contact misalignment. It is only when the oscillator is DC-coupled that V_0 can be measured.

Finally, the same germanium crystal was investigated using a Lock-in Amplifier. This is an extremely accurate device used to calculate an mean value of a time-varying voltage. It considers a given signal only – the one whereon it "locks in", inverts the polarity of the signal at a fixed point during each cycle, and then averages the resultant voltage over many cycles, thereby obtaining a mean value of the signal. The mean value depends on the fixed point at which polarity is reversed. Evidently, if polarity is reversed at points on a sinusoidal cycle where the signal is zero, then the average is simply the ordinary RMS value of the signal. However, if polarity is reversed at the maxima of cycles, then the resultant average is zero.

The foregoing property of the LIA was investigated with respect to the crystal of germanium: At a phase value of 7^o, the mean voltage (the mean of V_{34}, from the previous experiment) was observed to be equal to 12mV, and this was found to be a maximal value of voltage, since phases in the neighbourhood of 7^o gave smaller voltages. Introducing a phase shift of 90^o was found to change the mean voltage to zero.

However, the variation in mean voltages as a function of phase angle, in increments of 30^o, was not found to be significant, and so the graph of these data is omitted here.

The previous experiment (measuring V_{34} for many values of current through the germanium sample, with respect to the magic cylinder permanent magnet) was repeated. However, this time the resulting values are the *time-average* of V_{34}. The following values were obtained:

I / mA	$<V_{34}>$ / mV	$V_0=IR$ / mV	$V_H = <V_{34}> - V_0$ / mV
5	20.8	10.5	10.3
10	42.4	21.0	21.4
15	60.8	31.5	29.3

These values are not appreciably different from the values obtained using the DMM and the oscilloscope, and so the previous calculations are not repeated here.

Finally, to investigate the accuracy of the LIA, $<V_{34}>$ was measured for small currents, and the following results were obtained:

I / mA	$<V_{34}>$ / mV	$V_0=IR$ / mV	$V_H = <V_{34}> - V_0$ / mV	V_H / I / Ω
1	4.9	2.1	2.8	2.8
0.5	3.0	1.6	1.4	2.8
0.01	1.8	0.01	1.7	1.7

We see that V_H / I is not constant and moreover, the quantity V_H / IB does not give a value for α_H of the order of 12 V/TA. This is because it is difficult to obtaion precise measurements of $<V_{34}>$ for such small currents.

Conclusion:

The carrier concentrations for the first semiconductor sample was found to be $N = 4.0\times10^{20} \pm 0.2\times10^{20} / m^3$, while the conductivity was measured to be $\sigma = \left(4.65 \pm 0.25\times10^3\right)S$. The charge carriers are electrons. In the second case, the conductivity was observed to be $\sigma = 162 \pm 5S$, while the charge carriers were found to be holes. The ability of the lock-in amplifier to provide time-averaged voltages of a high precision was also noted. The Hall effect is thus shown to be a useful method of determining the electrical properties of materials.

(1) Shockley, William, *Electrons and Holes in Semiconductors*, (D. Van Nostrand Company Inc., 1950).